

Consumer Surplus and Consumer Welfare: Lecture VIII

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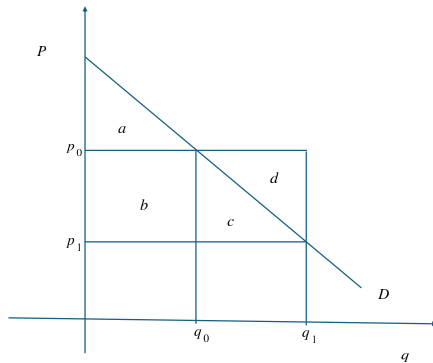
Overview

- The definition of a measure of the economic welfare of the consumer has been one of the most controversial subjects in economics.
- Unlike the producer case where profit provides an observable measure of surplus, we do not have an observable measure of consumer welfare.
- One concept is that the utility maximizing choice of consumption based on prices and income tell us something about consumer preference – we would assume that choosing at least one more of any good without changes in income would be preferred to the original point (non-satiation).
- Changes in the willingness-to-pay (WTP) on the part of consumers would provide some insight.
- Most measures do not seek to measure utility directly. Rather they estimate a revealed WTP in terms of money.

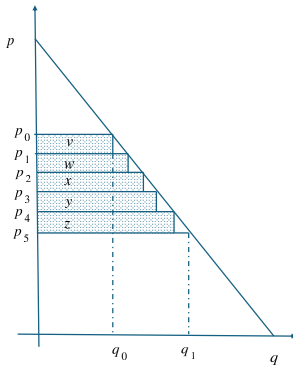
The Notion of Consumer Surplus

- Does it help to go back to the original definitions
 - **Compensating variation** is the amount of money which, when taken away from an individual after an economic change leaves the person as well off as before.
 - **Equivalent variation** is the amount of money paid to an individual which, if an economic change does not happen, leaves the individual just as well off as if the change had occurred.
- The scenario depicted in Figure 6 is a price decline from p_0 to p_1 which results in an increase from q_0 to q_1 .
- First, the decline from p_0 to p_1 holding the quantity constant means that the individual's expenditures would fall from $p_0 q_0$ to $p_1 q_0$ or $b = q_0 \times (p_0 - p_1)$ - *Equivalent variation*.

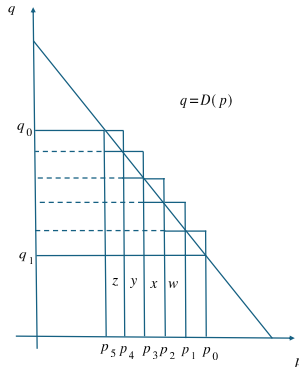
- Second, if we start from p_1 and increase the price to p_0 fixing the quantity at q_1 we would have to increase the individual's income by $(p_0 - p_1) \times q_1 = b + c + d$ to make the individual as well off – *Compensating variation*.



- These two are not equal. Is there anything we could do to make them equal?
- One way to make the two equal is to subdivide the price change into several smaller price changes as depicted in Panel (a) of Figure 8.
- One of my pet problems is that Marshall transposed the axes on the supply and demand curves - consider Figure Panel (b) of Figure 8



(a) Marshall



(a) Quantity a Function of Price

Defining Consumer Surplus

- As we discussed in Mathematical Statistics

$$\begin{aligned} A_1(N) &= \sum_{i=2}^N (p_i - p_{i-1}) \min_{q \in (q_i, q_{i-1})} D(q) \\ A_2(N) &= \sum_{i=2}^N (p_i - p_{i-1}) \max_{q \in (q_i, q_{i-1})} D(q) \end{aligned} \quad (1)$$

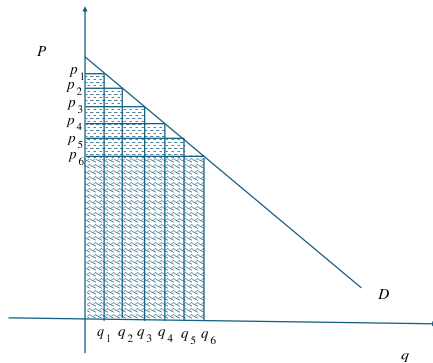
which define the Riemann sums.

$$\lim_{N \rightarrow \infty} A_1(N) = \lim_{N \rightarrow \infty} A_2(N) = \int_{p_N}^{p_0} D(p) dp. \quad (2)$$

- So as the number of slices becomes “large” we can define the consumer surplus as the area underneath the demand curve at the original price (p_0) above the lower price (p_1).

Willingness-To-Pay

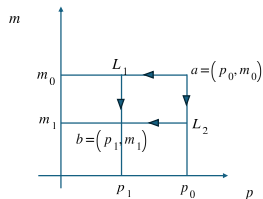
- Alternatively, it is the integral of the demand curve between p_0 and p_1 .
- Jules Dupuit (1844) who coined the phrase 'consumer surplus' also postulated that the consumer's demand curve was the maximum amount that the consumer was willing to pay for the last unit consumed.
- Hence, the demand curve also defines the consumer's marginal willingness-to-pay (WTP).
- Figure 11 uses this specification where at the first increment of the good, the consumer is willing to pay p_1 . However, for the second increment of good, the amount that the consumer is willing to pay falls to p_2 .



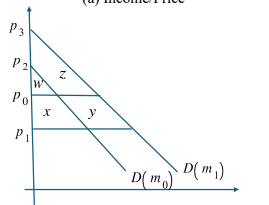
- In this context, the area below the demand curve is the *gross benefits* accruing to consuming q_6 units of the good.
- The economic 'net' economic benefit of this consumption is this area less the gross benefit forfeited from the income spent (the area $p_6 \times q_6$).
- Returning to Figure 6, following Dupuit, the consumer benefit would be the area $b + c$ because that is the area associated with the price decline from p_0 to p_1 .
- There are at least two reasons why the change in consumer surplus is an appealing measure of the benefit to consumers:
 - 1 it represents the sum of cost differences as price is continuously reduced from p_0 to p_1 , and
 - 2 it gives the change in what the consumer is willing to pay over that which is actually paid.

Path Dependence of Consumer Surplus

- It turns out that the change in consumer surplus is not so well defined for the case where several prices change simultaneously, or when income changes simultaneously with the price.
- The measure of consumer surplus is *path-dependent* in that it depends on whether the price changes first and the income, or whether income changes first and then the price.
- In Figure 14, we consider a change from point $a = (p_0, m_0)$ or the scenario where the price is p_0 and the income is m_0 to a new equilibrium $b = (p_1, m_1)$.
 - Panel a depicts the change in price/income space while panel b depicts the change in traditional demand space.
 - In panel b, the increase in income causes demand curve to shift out from $D(m_0)$ to $D(m_1)$.



(a) Income/Price



(b) Demand Curve

- In Figure 14, we start by considering the adjustment path L_1 where we reduce the price first and then adjust the income with the path L_2 where we adjust the income and then reduce the price.
- Following L_1 by decreasing the price first yields a consumer benefit of x based on the demand curve $D(m_0)$. To this area, we add the increase in income $m_1 - m_0$.
- Taking the second path L_2 , we first get the increase in income $m_1 - m_0$ to end up on the new demand curve $D(m_1)$. Given this change when we decrease the price from p_0 to p_1 we get a measure of $(m_1 - m_0) + x + y$.
- Hence, the same movement gives us two measure of Welfare.

Uniqueness of Consumer Surplus

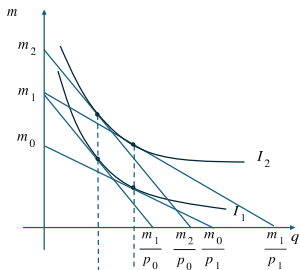
- The problem of nonuniqueness arises only in the context of *money measurement* of welfare change (in contrast to the change in utility which is unique).
- But *money measurement of welfare change is the only reasonable approach for most applied economic welfare problems because consumer utility is not directly observable.*
- It should also be emphasized that *even though a money measure is unique, it does not necessarily follow that it measures utility change.*
- An insight into the issue is that the two paths in Figure 14 (i.e., L_1 and L_2) are the same if and only if $D(m_0)$ and $D(m_1)$ coincide.
 - If the demand curve is not influenced by income changes, then the change in consumer surplus is uninfluenced by the path from (p_0, m_0) to (p_1, m_1) .

- When both price and income change *the consumer surplus measure is unique if and only if the income effect is zero.*
 - Basically, $\Delta q / \Delta m = 0$ - note that this does not imply a zero welfare effect.
 - This implies 'trivially' when the income elasticity is zero

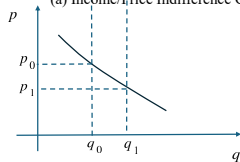
$$\eta = \frac{\Delta q}{\Delta m} \frac{m}{q} = 0 \quad (3)$$

- In my classes, we did this with a sledge hammer by assuming that we were dealing with Hicksian demand curves.

- The curve I_1 and I_2 in Figure 19 are the quantities that yield a fixed level of utility given an income/quantity combination.
- First, we start at income m_0 which increases to m_1 . While this seems a little backward - we are interested in the level q_1 at price p_1 .
- Working backward, we are interested in the income line $\overline{m_0 \frac{m_0}{p_1}}$ which is tangent to the isoquant I_1 .
- Using this isoquant - we want to determine the point of tangency between I_2 and the 'budget line' $\overline{m_1 \frac{m_1}{p_1}}$. Basically, we are defining a new price p_0 .
- Working back around, the point m_1 determines the q_1 at the tangency with I_2 .



(a) Income/Price Indifference Curves



(b) Demand Curve

- As income is changed from m_0 to m_1 at price p_1 the quantity remains q_1 .
 - Note that the 'budget lines' $\overline{m_0 \frac{m_0}{p_1}}$ and $\overline{m_1 \frac{m_1}{p_1}}$ are parallel. In addition, the points are directly above and below the other - the quantity is constant at q_1 - this determines the point (p_1, q_1) .
 - Also note that the 'budget lines' $\overline{m_1 \frac{m_1}{p_0}}$ and $\overline{m_2 \frac{m_2}{p_0}}$ are parallel. This determines the point q_0 .
- An interest implication can be developed by relating the conditions to a change in income.
 - To do this, one can use the concept of *zero-degree homogeneity of demand in price and income*.
 - This implies that a consumer's consumption bundle choice is not altered as the prices and income are adjusted proportionally.

- This condition is similar to the scenario where you redenominate a currency. Suppose that $p_1^1 = \alpha p_1^0$ and $p_2^1 = \alpha p_2^0$. The ratio of prices will remain unchanged

$$\frac{\Delta p_2}{\Delta p_1} = \frac{(\alpha - 1) p_2^0}{(\alpha - 1) p_1^0} = \frac{p_2^0}{p_1^0} \quad (4)$$

- Extending the concept to the tangency between the isoquant and the budget line

$$\frac{\Delta q_1}{\Delta q_2} = \frac{\Delta p_2}{\Delta p_1} = \frac{p_2^0}{p_1^0} \quad (5)$$

- The forgoing arguments also imply that the ratio of consumption adjustments in response to income levels is a constant determined by prices regardless of income level.
- Basically, the expansion paths are linear.

