

Welfare Measurement for the Producer – Part II: Lecture VII

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The Profit Maximizing Firm

- One issue in the application of welfare economics to agricultural policy is the fact that that we are often dealing with derived demand curves and not final demand curve.
 - The textbook provides a discussion of the application of welfare principles to the market for iron (or steel).
 - In our case, I will argue that we actually face a derived demand curve for agricultural output.
- As a starting point, consider the KLEM 35 data (Capital (K), Labor (L), Energy (E), and Materials (M)) data provided by the late Dale Jorgenson for 1960 through 2005.
 - This dataset provides output prices and quantities for 35 sectors (1 - Agriculture, Forestry, 2 - Metal mining, 3 - Coal mining, ...) and the input price and quantity for the four inputs.
 - Fortunately, the data also provides enough information to decompose the materials variable into agricultural and non-agricultural materials.

Derived Demand for Agricultural Inputs

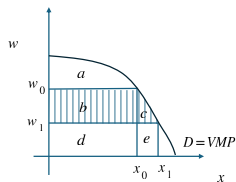
- One question is then - what the impact of the demand for agriculture based on some policy shift.
 - For example, what is the effect of the derived labor in the general economy on the such as an increase in the price of labor.
 - In this case, we will define a production relationship for industries that use agricultural materials as an input:
 - 1 Agriculture, Forestry, and fisheries
 - 2 Food and kindred products
 - 3 Textiles and apparels
 - 4 Wood and paper products
- Based on this I can specify a dual profit (or cost function)

$$\pi(\{p_1, p_2, p_3, p_4\} \{w_K, w_L, w_E, w_{MA}, w_{MO}\}) \quad (1)$$

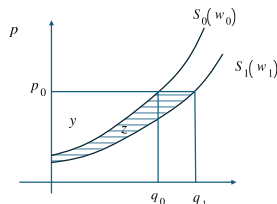
which can be used to derive a derived demand curve for agricultural production.

Input Price Changes in the Input Market

- Consider a single competitive firm (with fixed-product price) using a single input x .
- The input demand D for such a firm is determined as in Figure 6 by the value of marginal product (defined as output price multiplied by the marginal physical product of the input).
- The *compensating variation* of a fall in input price from w_0 to w_1 is the sum money the producer would be willing to pay to obtain the privilege of buying at the lower price (which would leave the firm just as well off at the lower price as if the price had not been lowered.)
- The *equivalent variation* is the sum of the money the producer would accept to forgo the privilege of buying at the lower price (which would leave the firm just as well off at the original price as if the price were lowered.)



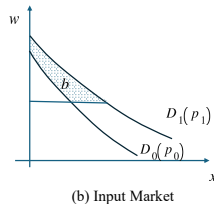
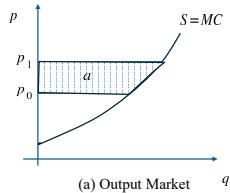
(a) Derived Demand Curve



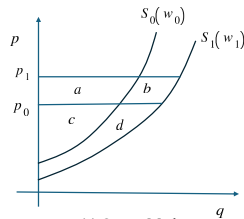
(b) Output Supply Curves

- Note that, at input price w_0 and corresponding profit-maximizing input use x_0 , the total cost is given by area $b + d$ (or $w_0 \times x_0$). and the total receipts are area $a + b + d$.
- Similarly, at price w_1 and input use x_1 , total variable cost is area $d + e$ and total receipts are area $a + b + c + d + e$.
- Thus, the shaded area $b + c$ measures the change in either quasirent or profit.
- Note that this shift can also be measured in output space – see panel (b) in the figure.
 - Basically, area y is the profit (and quasirent) before the change in input price, and $y + z$ is the profit after the change in price therefore $z = (y + z) - y$ is the compensating variation and quasirent.

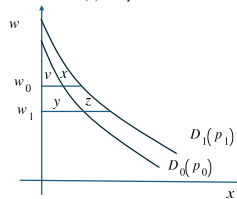
Output Price Changes Measure in Input Markets



Simultaneous Changes in Input and Output Prices



(a) Output Market



(b) Input Market

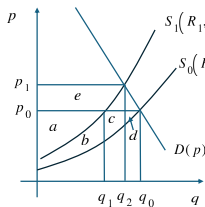
- In the output market (panel a) producer surplus is given by the area c at prices p_0 and w_0 , and by the area $a + b + c + d$ at prices p_1 and w_1 . The change in producer surplus is then $a + b + d$.
- In the input market (panel b) consumer surplus is the area $v + x + y + z$ at prices p_1 and w_1 compared with v with the prices given by p_0 and w_0 . The gain in consumer surplus is then $x + y + z$.
- Because both producer and consumer surpluses measure quasirents in the sing-input case, the change in quasirent from the multiple price change can be completely measured either in the output market ($a + b + d$) or the input market ($x + y + z$).

- The same analysis could be derived sequentially.
 - Moving from p_0 to p_1 results an increase of a in the output market.
 - Next, moving from w_0 to w_1 , the producer surplus increases by $b + d$.
 - Adding the two together gives the same result.
- Please look at the case of multiple inputs.

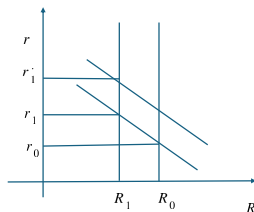
Input Quantity Restrictions for the Competitive Firm

- Agricultural policy has experienced several variations since the Agricultural Adjustment Act of 1933.
- The original act created allotments (acres of land for each crop allocated to each farm). The concept was to increase the price by reducing the overall supply of each “program crop.”
- As depicted in Figure 13, the movement to allotments represents an upward shift from $S_0(R_0, x_2(R_0))$ to $S_1(R_1, x_2(R_1))$.
- If we hold the price constant, this reduces the level of output to q_1 .
 - The results for this scenario involve the production relationship between the inputs. If the inputs are complements, the derived demand curve simply shift backward (as in the text).
 - However, if the inputs are substitutes, decreasing the level of R causes the input demand curve to shift backward and rotate.

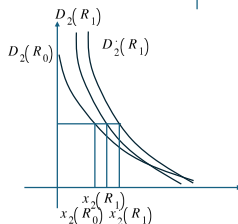
The Effect of Allotments



(a) Output Market



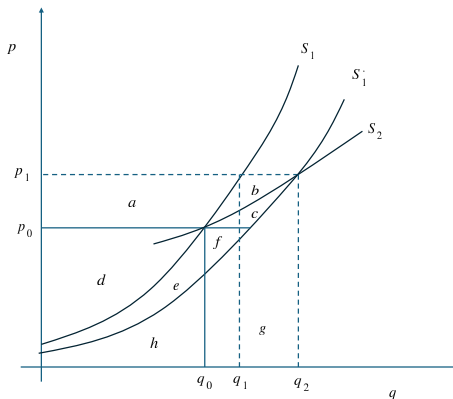
(b) Land Market



(c) Labor Market

- This rotation leads to something called “slippage” – the reduction in the supply is smaller because the labor use per acre increases.
- The change in producer surplus is $b + c + d$.
- Of course, we neglect the benefits to the program – the price increases from p_0 to p_1 which results in an increase in producer surplus of e .
- Of course, we are not considering the loss in consumer welfare.

Investment and Intertemporal Measurement



Integrability and Unique Measurement in Practice

- We have been dealing with models with increasing complexity.
 - We started with an empirical model of a single market and “solved” the system using the current market equilibrium and an estimated elasticity.
 - In this and the last lecture, we have moved to a system of outputs and inputs.
 - Implicitly, these curves have to “make sense.”
- For the purpose of this class, we will be primarily interested in the regularity conditions implied by duality.

- Starting with a multivariate primal

$$y = \alpha_0 + \alpha'x + \frac{1}{2}x'Ax. \quad (2)$$

For the economic decision to make sense, this representation has to be increasing and concave in each input.

- If these conditions are met, the output supply curves slope upward and input demand curves slope downward.
- Further there are several cross equation restrictions – typically $\partial^2 y / \partial x_1 \partial x_2 = \partial^2 y / \partial x_2 \partial x_1$ and the implied second derivative matrix is negative semi-definite. These properties are required for a maximum profit to exist.

- Moving from the primal to the dual

$$\max_x p'y - w'x \text{ s.t. } y = f(x) \Rightarrow \pi(p, w) \quad (3)$$

- This transformation also implies a host of regularity conditions which imply that the system of supply and demand equations integrate (i.e., the profit function is convex in output prices and concave in input prices).