

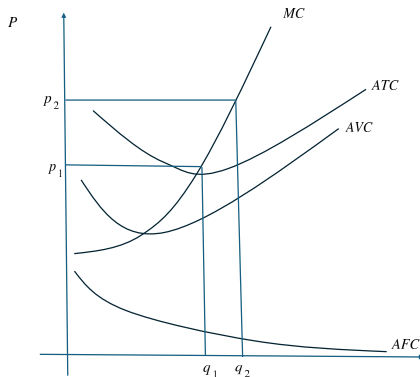
Welfare Measurement for the Producer – Part I: Lecture VI

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 - Profit
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The Profit Maximizing Firm



- The basic framework for measurement of welfare for producers is the neoclassical model (Marshall).
- Following the simple economic model as presented in Figure 3, the producer chooses to produce at the point where the price of the output is equal to the marginal cost of production.
 - As depicted in Figure 3, if the price of output increases from p_0 to p_1 , the producer increases the level of output produced from q_0 to q_1 .
 - *Is the producer better off, and by how much?*
 - Put another way, has there been an increase in the revenue that can be 'taxed away' without making the firm any worse off due to the price change?
- There are questions that will be slightly different. Specifically, the change in Figure 3 represents a small change. Does the measurement change if a decrease in price leads the firm to cease production?

Introduction to Production Economics

- The profit function

$$\pi(p, y) = \sum_{i=1}^N p_i(q) q_i - c(w, q) - FC \quad (1)$$

where p_i is the output price for output i , q_i is the level of output i produced, w_i is the price of the i^{th} input, FC is the level of fixed cost, and not explicitly in Equation 1, x_i is the level of input i used in production.

- First, Equation 1 allows for the possibility of market power. This will be an emerging issue in our analysis (i.e., what is the economic cost of a monopoly or monopsony).

- Holding output prices constant, the first-order conditions of Equation 1 reproduces the results from Figure 3.

$$\frac{\partial \pi(p, x)}{\partial q_i} = p_i - \frac{\partial c(w, q)}{\partial q_i} \quad (2)$$

- In AEB 7184, we develop an empirical representation for the profit function and its regularity conditions. One simple form of this function is a Translog profit function

$$\begin{aligned} \ln(\pi) = & \alpha_0 + \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}' \begin{bmatrix} \ln(p_1) \\ \ln(p_2) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \ln(p_1) \\ \ln(p_2) \end{bmatrix}' \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} \ln(p_1) \\ \ln(p_2) \end{bmatrix} + \\ & \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \begin{bmatrix} \ln(w_1) \\ \ln(w_2) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \ln(w_1) \\ \ln(w_2) \end{bmatrix}' \begin{bmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{bmatrix} \begin{bmatrix} \ln(w_1) \\ \ln(w_2) \end{bmatrix} + \\ & \begin{bmatrix} \ln(p_1) \\ \ln(p_2) \end{bmatrix}' \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \ln(w_1) \\ \ln(w_2) \end{bmatrix} \end{aligned} \quad (3)$$

- By Hotelling's lemma we have

$$\begin{aligned}\frac{\partial \pi(p, w)}{\partial p_i} &= q_i(p, w) \\ \frac{\partial \pi(p, w)}{\partial w_j} &= x_j(p, w)\end{aligned}\tag{4}$$

- A primary question is whether the input demand function is conditioned on the level of output or unconditioned. The input demand curves in Equation 4 are unconditional.
- The conditional input demand curve is derived by applying Sheppard's lemma to the cost function

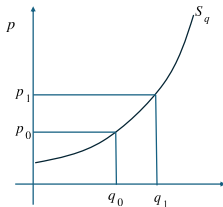
$$\frac{\partial c(w, q)}{\partial w_i} = \tilde{x}_i(w, q)\tag{5}$$

- As a starting point we consider the scenario where the firm faces an increase in the price of output from p_0 to p_1 .
- Panel (a) presents the standard output choice model presented since freshman economics (output increases from q_0 to q_1 .)
- Panel (b) gives the change in rents to quasi-fixed factors of production such as owner labor and land.
 - To develop this concept, we extend the profit function slightly to include terms with that involve $R - \pi(p, w, R)$.
 - We can then derive the shadow value these quasi-fixed resources as

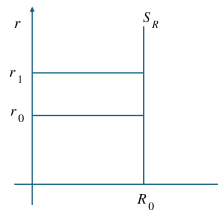
$$\frac{\partial \pi(p, w, R)}{\partial R_k} = r_0 \quad (6)$$

at the original point of production.

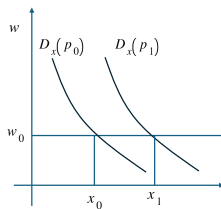
- Given that the quantity of these quasi-fixed inputs are fixed in the short-run, the supply function for these inputs is perfectly inelastic S_R .



(a) Output Choice



(b) Rents to Fixed Factors



(c) Input Demand

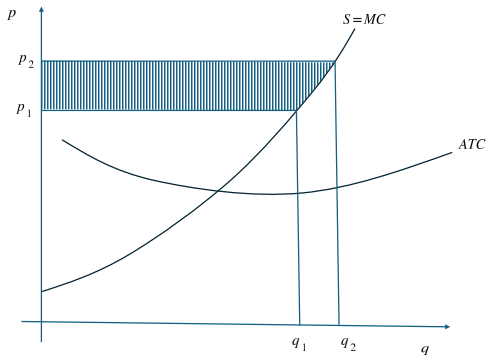
- As the firm expands its output, the demand for the variable input x increases from $D_x(p_0)$ to $D_x(p_1)$ leading to an increase in the amount used by the firm from x_0 to x_1 .

Profit

- Profit is the most obvious candidate for a measure of producer welfare because at this point, maximizing profit is the assumed objective of the firm

$$\pi = TR - TC = TR - TVC - TFC. \quad (7)$$

- To what extent does profit measure producer welfare?
 - To determine the relationship between profit and producer welfare, we start by reviewing the compensating and equivalent variation.
 - In addition, we will start with the assumption that the firm produces a positive output before and after the change.
- In Figure 12, we assume that the price increases from p_1 to p_2 .



- *The compensating variation associated with the price increase is the sum of money that, when taken away from the producing firm leaves it just as well off as if the price did not change, given that it is free to adjust production (to profit-maximizing quantities) in either case.*
 - Because profit at p_2 (and q_2) is higher than at p_1 (and q_1) the shaded area is exactly the compensating variation.
- *The equivalent variation associated with the price increase is the sum of money which, when given to the firm, leaves it just as well off without the price change as if the change had occurred, again assuming freedom of adjustment.*
 - Because profit at p_1 is lower than at p_2 by the amount of the shaded area, this area must also represent the equivalent variation.
- Note that in this case the change in profit provides the same measure for the compensating variation and the equivalent variation.

Rent

- As an alternative to profit Alfred Marshall (1930) defined a producer's net benefit as the excess of gross receipts which a producer receives for any commodity produced over their prime cost – that is, over the extra cost that the firm incurs in order to produce those things which it could have escaped if it had not produced them.
 - Currie, J.M., Murphy, J.A. & Schmitz, A. (1971) The Concept of Economic Surplus and Its Use in Economic Analysis *Economic Journal* 81(324), 741-799.
- This concept has been called quasirent for the firm because it is a rent on fixed factors employed by the firm but, unlike factor rent, may not persist over a long period of time.

- *Quasirent R is defined as the excess gross receipts TR over total variable cost TVC , $R = TR - TVC$.*
- Marshall went on to suggest the area below the price line and above the supply curve – commonly called producer surplus – as a measure of this benefit.
- *Producers surplus is defined as the area above the supply curve and below the price line of a corresponding firm or industry.*
 - Stigler, G.J. (1952) *The theory of Price* New York: Macmillan.
- Letting R be quasirent and P be producer surplus

$$P = R = TR - TVC \quad (8)$$

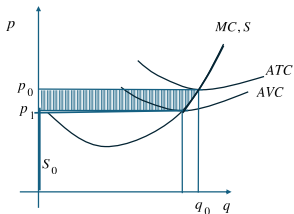
That is, the area above a competitive firm's short-run supply curve and below the price line provides a measure of the 'excess gross receipts over prime costs' because the firm's short-run supply curve coincides with its marginal cost curve above AVC .

- Based on the analysis in Figure 12

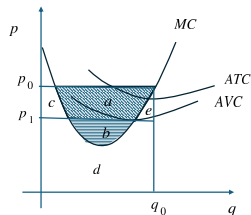
$$R = P = \pi + TFC \quad (9)$$

or that both quasirent and producer surplus are given by profit plus total fixed cost.

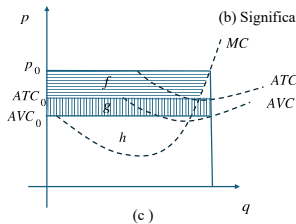
The Relationship of Profit, Quasirents, and Producer Surplus



(a) Shutdown



(b) Significant Initial Economies



(c)

- Panel (a) shows the effect of a shut down, for any price less than p_1 , the firm shuts down and does not produce any output.
 - Profit and quasirents are the same – Producer Surplus
- Panel (b) suggests a model of quasi-rents. At price p_0 , the profit-maximizing total receipts are given by the area below p_0 and left of quantity q_0

$$TR = a + b + c + d + e = p_0 \times q_0. \quad (10)$$

Here the total variable cost is the area below the total variable cost curve

$$TVC = c + d + e \quad (11)$$

The quasirent is then computed as

$$R = TR - TVC = a + b. \quad (12)$$

- In panel (c) we are again dealing with the area below p_0 and left of the quantity q_0

$$TR = f + g + h = p_0 \times q_0 \quad (13)$$

Note that

$$\begin{aligned} TVC &= h = AVC \times q_0 \\ \Rightarrow R &= f + g = (p_0 - AVC_0) \times q_0 \end{aligned} \quad (14)$$