

Compensation Principle and the Welfare Function - Part II: Lecture V

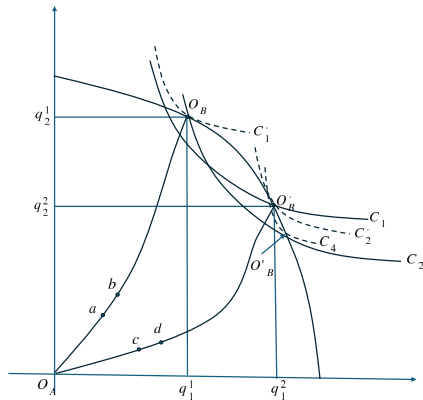
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Social Welfare Function

- Returning to some notions we developed in Lecture IV, we developed the Pareto efficient set of utility combinations $\{U_A, U_B\} - UU$
- Intuitively, we cannot choose a point along this curve without some knowledge - this knowledge is referred to as the Social Welfare Function.
- Basically, the function we used in Lecture III to define the Scitovsky Indifference Curve was a Social Welfare Function.
- In Figure 4, W can be thought of as our function $U_A^\psi U_B^{1-\psi}$ – it reduces this frontier to a single point.
- The tangency between W and UU is sometimes called the *fourth optimality condition*.



- The assumption that the social welfare function is determined by the utilities of all individuals has been called the *fundamental ethical postulate* by Samuelson (1947) and is the cornerstone of democratic societies.
- Bergson, A. (1938). A Reformulation of Certain Aspects of Welfare Economics *Quarterly Journal of Economics* 52(2), 310-314.

- We would expect that this Welfare function would follow properties similar to the utility function.
 - 1 An increase in the utility of any individual holding others constant increases social welfare (the Pareto principle);
 - 2 If one individual is made worse off, then another individual must be made better off to retain the same level of social welfare; and
 - 3 If some individual has a very high level of utility and another has a very low level of utility, then society is willing to give up some of the former individual's utility to obtain even a somewhat smaller increase in the latter individual's utility, with the intensity of the trade-off depending upon the degree of inequality.

Limitations of the Social Welfare Function Approach

- Although a social welfare function is a convenient and powerful concept in theory, its practical usefulness has been illusory.
- The major approaches that have been attempted include (1) the subjective approach, (2) the basic axiomatic approach, and (3) the moral approach.
- The subjective approach is represented by those who postulate a complete functional form for social welfare function on ethical grounds. Starting with Bentham (1961/1823) who conceptualized that changes in happiness should simply be added over individuals

$$W = U_1 + U_2 + U_3 + \dots \quad (1)$$

Note that the slope of the welfare at optimum is -1 for this welfare function.

- Another possibility of the subjective approach rejects the “linearity” of Equation 1 because it does not recognize income inequality.
- An alternative (similar to the “power utility function” applied in risk analysis) explicitly adjusts for income inequality

$$W = \frac{1}{1-\rho} \left[U_1^{1-\rho} + U_2^{1-\rho} + \dots \right] \quad (2)$$

- The axiomatic approach, on the other hand, attempts to investigate the existence and form of the social welfare function mathematically based on a set of plausible underlying axioms about individual preferences and how they count to society.

- The most celebrated of these efforts is Arrows (1951) *impossibility theorem*
- Arrow, K.J. (2020) *Social Choice and Individual Values* Barakaldo Books.
- Arrow showed that no such rule existed under the following plausible requirements:
 - 1 The domain of decisions is unrestricted.
 - 2 The Pareto principle applies.
 - 3 Dictatorship is ruled out.
 - 4 Rankings are independent of irrelevant alternatives.

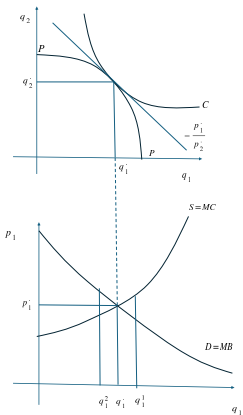
- The moral justice approach argues that basic axiomatic examination following Arrow fail because majority groups acting selfishly will prefer to eliminate consideration of minority interests.
- Harsanyi, J.C. (1953) Cardinal Utility in Welfare Economics and in the Theory of Risk-Taking *Journal of Political Economy* 61(5), 434-435.
- Harsanyi, J.C. (1955) Cardinal Utility, Individual Ethics and Interpersonal Comparisons of Utility *Journal of Political Economy* 63(4), 309-321.
- Under a relatively weak set of assumptions, Harsanyi shows with this approach that the social welfare function is a weighted sum of individual utilities

$$W = \alpha_1 U_1 + \alpha_2 U_2 + \alpha_3 U_3 + \cdots . \quad (3)$$

- Rawls, J. (1999) *A Theory of Justice* Revised Edition, Harvard University Press.
- This theory, which is really more of a philosophy than a theory, contends that policy should be evaluated by the welfare of the most miserable person in society.

$$W = \min (U_1, U_2, \dots) \quad (4)$$

Practical Applied Policy Analysis: The Relationship of General Equilibrium and Partial Equilibrium Analysis



- The top graph is the general equilibrium analysis that we have been dealing with.
 - The optimal amount of $\{q_1, q_2\}$ is determined by the tangency between the production possibility frontier (PP) and the SIC curve C .
 - Economically, this point is determined by the price ratio $-p_1/p_2$.
- Following this down to the bottom graph, this intersection determines and is determined by the intersection between the supply (or marginal cost of production) of q_1 and the demand (or the marginal benefit) of q_1 .
 - Working from the bottom curve up, if we start at q_1^2 , we would improve societal welfare in the top graph by increasing the amount of q_1 produced (and decreasing the amount of q_2 produced).
 - Starting from the other side, if we start at q_1^1 , we would improve societal welfare in the top graph by decreasing the amount of q_1 produced (and increasing the amount of q_2 produced.)