

Compensation Principle and the Welfare Function - Part I: Lecture IV

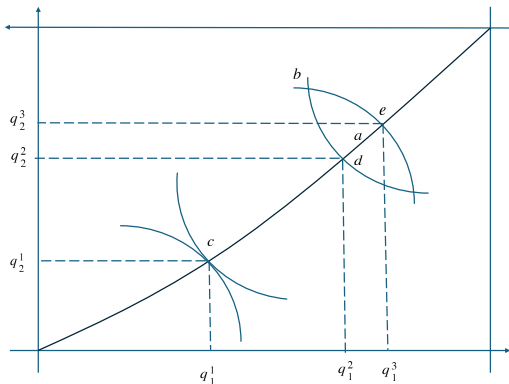
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- 1 The Compensation Principle
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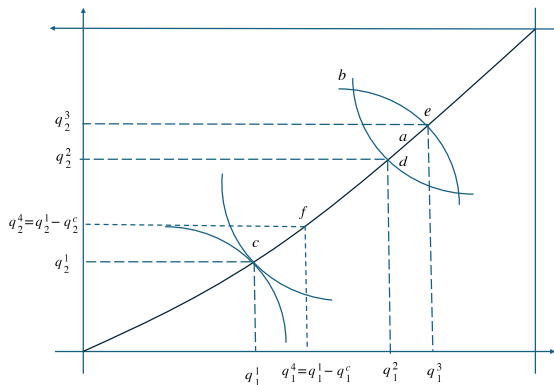
- 2 Utility Possibility Curves and the Potential Welfare Criteria

The Pure Consumption Case



- In figure 3, point a is preferred to point b on the basis of the Pareto principle.
 - Basically, any point inside the “lemon” or “cigar” shaped area is a Pareto improving move.
 - As we discussed in previous lectures, point a is on the contract curve and represents a Pareto optimal location.
- But how does one compare point b with a point such as c , where c is not inside the lens-shaped area?
 - The question is whether we can create a “compensating” change such that by paying B paying A at least one of the individuals can be made better off without making the other worse off.
 - The point is that the movement from d to c creates an “excess” amount of each good (the excess of $q_1 = q_1^2 - q_1^1$ and $q_2 = q_2^2 - q_2^1$)
 - The basic concept is that individual B can create a payment $(\{q_1^c, q_2^c\})$ such that both individuals are better off.

Compensating Movement



Compensation in Consumption Case – Details

- For the compensating solution to exist

$$U_A(q_1^1 + q_1^c, q_2^1 + q_2^c) \geq U_A(q_1^2, q_2^2) \quad (1)$$

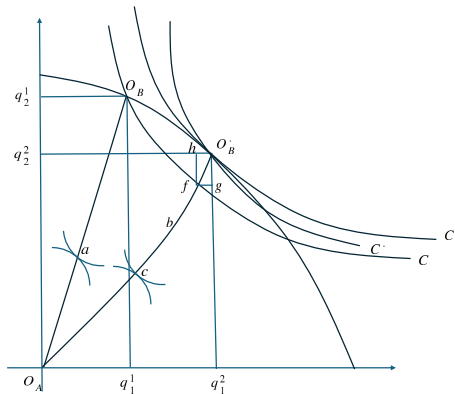
and

$$U_B(q_T - (q_1^1 - q_1^c), q_2^T - (q_2^1 - q_2^c)) \geq U_B(q_1^2, q_2^2) \quad (2)$$

- By the compensation principle, every-one is potentially better off by moving from point b to point c because the amount individual B gains is greater than the amount that individual A loses.*
- This result holds even if the compensation is not paid.

Distribution of Different Bundles

- Next, we are going to use the full implications of the Scitovsky Indifference Curves (*SIC*) from Lecture III.
- As presented in Figure 8, picture the movement from one point on the production possibilities curve (O_B) to another (O'_B).
- We will assume that O_B is not optimal in the sense that the production possibilities frontier is not tangent to the *SIC* at this point.
- What we want to analyze is whether it is possible to move from one solution to the other using the compensation principle.



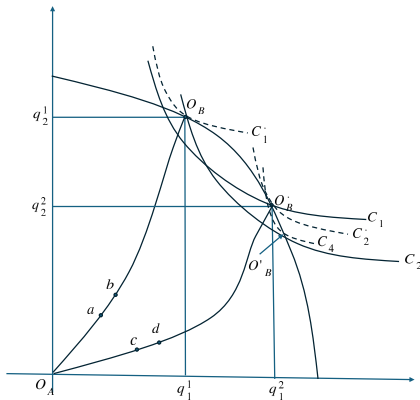
- First, we assume that point a is the point on the contract curve associated with the tangency for SIC C and b is the point on the contract curve associated with the tangency of SIC curve C' .
- The point does the compensation principle support moving from O_B to O_B^* ?
 - Note that at point b , individual B is worse off - he receives less of both goods.
 - On the flip-side individual A is better off.
- The point is that the second equilibrium implies more of both goods (i.e., $\Delta q_2 = fh$ and $\Delta q_1 = fg$).

- Using the compensation principle with initial production bundle at O_B and distribution point a , a move to the production bundle at O_B^* is supported regardless of the way that it is distributed.
- Using the Pareto principle, however, the move is supported only if the actual distribution corresponds to moving along the Scitovsky curve C to point f , keeping the welfare of each individual constant and then dividing the excess of fg of good q_1 and fh of good q_2 among the two individuals in some way that neither is worse off.

The Reversal Paradox

- For the case where the gainers can potentially compensate the losers, a conclusion that one position is better than another is not warranted.
 - *One must ask, also, whether the losers can bribe the gainers not to make the move.*
- In Figure 12, we begin with two points O_B and O_B^* . We assume that the two *SIC* curves that run through these points are C_1 and C_2 , respectively.
- The 'standard' question is whether moving from O_B to O_B^* meets the compensation principle.

Scitovsky Paradox



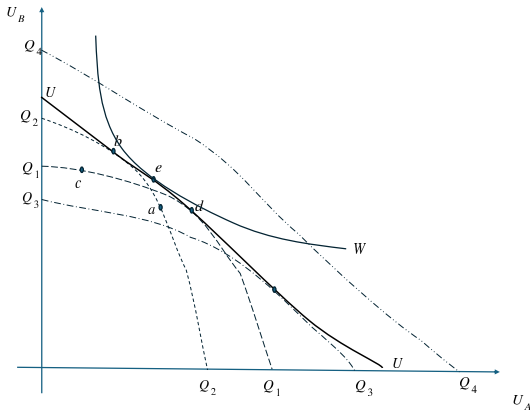
- If the movement supports the compensation principle - assume that point c is better than a .
- Note, however, that this implies a movement from C_1 to C_2 - the two points are on different SIC curves.
- However, by the compensation principle, O_B^* is only potentially better off.
- In addition, neither SIC curve is optimal.
 - For O_B , the optimal SIC curve is C'_1 which implies the allocation at point b .
 - For O'_B , the optimal SIC curve is C'_2 which implies an allocation of d .
- Because compensation is not actually paid, a reversal problem arises.
 - That is, at the new production O_B^* , point c is a *second best* solution - point d is preferred to point c .

- Thus, it is possible that some point on the original production envelope - say b is preferred to point d .
- This movement back to the original production is referred to as the *Scitovsky reversal*.
- Other points.
 - *The reversal paradox occurs only in comparing two second-best bundles.* It does not arise if one of the bundles is first-best.
 - *The reversal does not always occur in comparing two second-best bundles even though compensation is not actually paid.*

Utility Possibility Curves and the Potential Welfare Criteria

- First, a slight restatement of the *SIC* curves. In Lecture III, I used a social utility curve $F(U_1, U_2)$ to derive an mathematical specification of the Scitovsky Indifference Curves. Note that this curve exists regardless of whether I can write down this social welfare function.
- Basically, the utility surface (collection of isoquants) exist whether I can specify (or know) the mathematical specification of the curves.
 - We use mathematical functions such as the Cobb-Douglas function to derive mathematical results which prove our point.
 - However, the collection of isoquants exist, individuals can rank bundles. It is this ranking that is important.
 - In our ordinalist world, we cannot know the actual functions.
- Now suppose that we abstract away from my simple welfare function.

Utility Possibility Curves



- Figure 16 presents the utility of each individual (U_A, U_B) along the contract curve. So Q_1Q_1 are the combinations of utilities generated by a fixed output bundle O_B .
 - Note that the different points on the Q_1Q_1 curve implies a different level of individual utilities $\{U_A, U_B\}$ – a different point on the contract curve.
- Note that we typically go to a “squiggly” line implying that we do not know the exact shape (i.e., we do not know whether this surface is smooth – locally constant C^2).
- Conceptually, we can join a locus of points representing the outer envelope tangent to each curve (U) – called the *grand utility possibility frontier* UU . These points are *First-Best solutions*.
- Note that the different contract curves (Q_1Q_1, Q_2Q_2, Q_3Q_3) come from a single production possibilities curve.
 - The Q_4Q_4 represents an outward shift in the production possibilities curve.