

Scitovsky Indifference Curves: Lecture III

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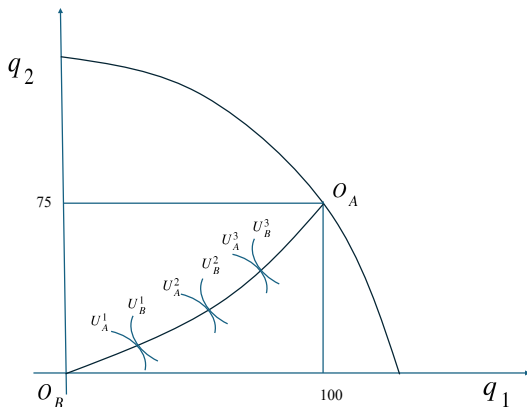
March 3, 2026

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Scitovsky Indifference Curves

- In Lecture II, we introduced the Scitovszky Indifference Curves (*SIC*) and how it is tangent to the *Production Possibilities Frontier* at some optimal point.
- It is worth a little time to more fully develop the *SIC* and how it fits into welfare analysis.
- As a starting point, consider the 'Inscribed' Edgeworth-Bowley depicted in Figure 4.
- In this figure the locus of points O_BO_A is referred to as the contract curve, it traces out those points where the Marginal Rate of Substitution between good 1 and good 2 are the same for both consumers (i.e., consumer *A* and consumer *B*).

Inscribed Edgeworth-Bowley Box



Contract Curve for the Cobb-Douglas Utility Function

- In order to develop the Scitovsky function, we consider the maximization problem using a simple Cobb-Douglas utility function

$$\begin{aligned} L &= q_1^\alpha q_2^\beta + \lambda (Y - p_1 q_1 - p_2 q_2) \\ \frac{\partial L}{\partial q_1} &= \alpha \frac{U}{q_1} - \lambda p_1 = 0 \\ \frac{\partial L}{\partial q_2} &= \beta \frac{U}{q_2} - \lambda p_2 = 0 \\ \frac{\partial L}{\partial \lambda} &= Y - p_1 q_1 - p_2 q_2 = 0 \end{aligned} \tag{1}$$

- A standard problem is to use the first-order conditions in Equation 1 to solve for the Marshallian demand curves

$$q_1 = \frac{\alpha}{\alpha + \beta} \frac{Y}{p_1}, \quad q_2 = \frac{\beta}{\alpha + \beta} \frac{Y}{p_2} \quad (2)$$

Contract Curve for the Cobb-Douglas Function (II)

- However, in this applications we take a little different approach. Specifically, note that in Figure 4 we are interested in those points where the Marginal Rate of Substitution is equal for each consumer.
- Hence, we start by using the results in Equation 1 to solve for the Marginal Rate of Substitution

$$\frac{\frac{\partial L}{\partial q_2}}{\frac{\partial L}{\partial q_1}} \Rightarrow \frac{\partial q_1}{\partial q_2} = \frac{\beta q_1}{\alpha q_2} \quad (3)$$

Contract Curve for the Cobb-Douglas Function (III)

- Denoting q_1^A and q_2^A as the consumption for consumer A and q_1^B and q_2^B as the consumption for consumer B , we can solve for the contract curve by solving

$$\frac{\beta_A q_1^A}{\alpha_A q_2^A} = \frac{p_2}{p_1} = \frac{\beta_B q_1^B}{\alpha_B q_2^B} \quad (4)$$

- Next, consider imposing (implicitly) the constraint that the amount consumed has to be somewhere within the Edgeworth-Bowley box

$$\begin{aligned} q_1^T &= q_1^A + q_1^B \\ q_2^T &= q_2^A + q_2^B \end{aligned} \quad (5)$$

Contract Curve for the Cobb-Douglas Function (IV)

- Hence, we can rewrite Equation 4 as

$$\frac{\beta_A q_1^A}{\alpha_A q_2^A} = \frac{\beta_B (q_1^T - q_1^A)}{\alpha_B (q_2^T - q_2^A)}. \quad (6)$$

- Solving Equation 6 yields

$$q_1^A = \frac{K q_1^T q_2^A}{q_2^T + (K - 1) q_2^A}.$$

$$K = \frac{\beta_B \alpha_A}{\alpha_B \beta_A}.$$
(7)

Hence, by selecting q_2^A Equation 7 yields the level of q_1^A .
 Then putting this result into Equation 5 yields q_1^B and q_2^B .
 And assuming that $\alpha_A = 0.4$, $\beta_A = 0.6$, $\alpha_B = 0.6$, $\beta_B = 0.4$,
 $q_1^T = 100$, and $q_2^T = 75$.

Solutions for the Contract Curve

q_1^1	q_2^1	q_1^2	q_2^2	MRS_1	MRS_2	U_1	U_2	$F(U_1, U_2)$
6.400	10.000	93.600	65.000	0.960	0.960	8.365	80.897	3.25862
8.163	12.500	91.837	62.500	0.980	0.980	10.541	78.734	3.36068
10.000	15.000	90.000	60.000	1.000	1.000	12.754	76.525	3.44174
11.915	17.500	88.085	57.500	1.021	1.021	15.006	74.269	3.50807
13.913	20.000	86.087	55.000	1.043	1.043	17.298	71.963	3.56336
16.000	22.500	84.000	52.500	1.067	1.067	19.632	69.604	3.60998
18.182	25.000	81.818	50.000	1.091	1.091	22.010	67.189	3.64950
20.465	27.500	79.535	47.500	1.116	1.116	24.435	64.716	3.68300
22.857	30.000	77.143	45.000	1.143	1.143	26.908	62.182	3.71124
25.366	32.500	74.634	42.500	1.171	1.171	29.433	59.582	3.73473
28.000	35.000	72.000	40.000	1.200	1.200	32.011	56.915	3.75382
30.769	37.500	69.231	37.500	1.231	1.231	34.647	54.174	3.76871
33.684	40.000	66.316	35.000	1.263	1.263	37.343	51.357	3.77947
36.757	42.500	63.243	32.500	1.297	1.297	40.102	48.458	3.78606
40.000	45.000	60.000	30.000	1.333	1.333	42.929	45.471	3.78832
43.429	47.500	56.571	27.500	1.371	1.371	45.828	42.393	3.78593
47.059	50.000	52.941	25.000	1.412	1.412	48.802	39.215	3.77842
50.909	52.500	49.091	22.500	1.455	1.455	51.858	35.931	3.76506
55.000	55.000	45.000	20.000	1.500	1.500	55.000	32.534	3.74481
59.355	57.500	40.645	17.500	1.548	1.548	58.235	29.015	3.71615
64.000	60.000	36.000	15.000	1.600	1.600	61.569	25.364	3.67675
68.966	62.500	31.034	12.500	1.655	1.655	65.010	21.571	3.62295
74.286	65.000	25.714	10.000	1.714	1.714	68.566	17.624	3.54853
80.000	67.500	20.000	7.500	1.778	1.778	72.247	13.510	3.44174
86.154	70.000	13.846	5.000	1.846	1.846	76.062	9.213	3.27607
92.800	72.500	7.200	2.500	1.920	1.920	80.024	4.716	2.96665

Social Welfare Curve

- In this case, we assume a social utility curve

$$F(U_A, U_B) = \psi \ln(U_A) + (1 - \psi) \ln(U_B). \quad (8)$$

where in this application $\psi = 0.5$. Note that this curve reaches a maximum at $U_A = 42.929$ and $U_B = 45.471$ where $F(U_A, U_B) = 3.788$. This is the supreme of social welfare given this social welfare function.

- Drilling down into this concept, for any q_1^A the social welfare function can be written as

$$F = F(U_A(q_1^A(q_2^A), q_2^A), U_B(q_1^B(q_2^A, q_1^T), q_2^B(q_2^A, q_2^T))) \quad (9)$$

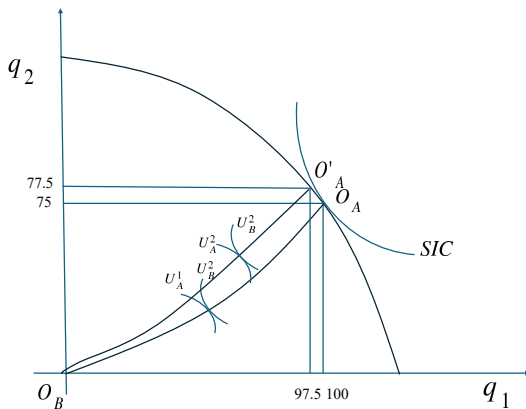
where $q_2^1(q_1^1)$ is determined by Equation 7 and $q_1^B = q_1^T - q_1^A$ and $q_2^B = q_2^T - q_2^A$ (the summing up restriction inside the Edgeworth-Bowley box).

A Mathematical Definition of the *SIC*

- We can conceptualize choosing the value of q_1^A that maximizes Equation 9, to rewrite

$$\begin{aligned} \max_{q_2^A} F \left(U_A \left(q_1^A \left(q_2^A \right), q_2^A \right), U_B \left(q_1^B \left(q_2^A, q_1^T \right), q_2^B \left(q_2^A, q_2^T \right) \right) \right) \\ = \tilde{F} \left(q_1^T, q_2^T \right) \end{aligned} \quad (10)$$

- This is a single point on Figure 4. Next, we consider running a curve through this point.
- Next, we consider changing the dimensions of the box as depicted in Figure 13.
- As a starting point, we consider changing to $q_1^T = 97.5$ and $q_2^T = 77.5$.



Changed Size of Edgeworth-Bowley Box

q_1^1	q_2^1	q_1^2	q_2^2	MRS_1	MRS_2	U_1	U_2	$F(U_1, U_2)$
6.023	10.000	91.477	67.500	0.903	0.903	8.165	81.004	3.24715
7.677	12.500	92.323	62.500	0.921	0.985	10.286	78.984	3.34999
9.398	15.000	90.602	60.000	0.940	1.007	12.441	76.832	3.43132
11.189	17.500	88.811	57.500	0.959	1.030	14.633	74.636	3.49795
13.054	20.000	86.946	55.000	0.979	1.054	16.862	72.393	3.55360
15.000	22.500	85.000	52.500	1.000	1.079	19.131	70.100	3.60062
17.031	25.000	82.969	50.000	1.022	1.106	21.442	67.755	3.64061
19.152	27.500	80.848	47.500	1.045	1.135	23.795	65.355	3.67465
21.370	30.000	78.630	45.000	1.068	1.165	26.193	62.898	3.70351
23.692	32.500	76.308	42.500	1.093	1.197	28.640	60.381	3.72773
26.124	35.000	73.876	40.000	1.120	1.231	31.136	57.800	3.74767
28.676	37.500	71.324	37.500	1.147	1.268	33.684	55.151	3.76355
31.357	40.000	68.643	35.000	1.176	1.307	36.288	52.431	3.77550
34.175	42.500	65.825	32.500	1.206	1.350	38.951	49.635	3.78350
37.143	45.000	62.857	30.000	1.238	1.397	41.675	46.759	3.78745
40.272	47.500	59.728	27.500	1.272	1.448	44.465	43.797	3.78713
43.575	50.000	56.425	25.000	1.307	1.505	47.324	40.743	3.78215
47.069	52.500	52.931	22.500	1.345	1.568	50.256	37.592	3.77197
50.769	55.000	49.231	20.000	1.385	1.641	53.267	34.336	3.75576
54.695	57.500	45.305	17.500	1.427	1.726	56.361	30.967	3.73235
58.868	60.000	41.132	15.000	1.472	1.828	59.545	27.475	3.70001
63.312	62.500	36.688	12.500	1.519	1.957	62.823	23.850	3.65605
68.054	65.000	31.946	10.000	1.570	2.130	66.205	20.075	3.59611
73.125	67.500	26.875	7.500	1.625	2.389	69.696	16.130	3.51241
78.561	70.000	21.439	5.000	1.683	2.859	73.306	11.976	3.38877
84.403	72.500	15.597	2.500	1.746	4.159	77.045	7.499	3.17958

- I will admit that the difference is small, but optimal point in Table 14 is a little smaller than the optimal value in Table 10 ($3.78832 - 3.78745 = 0.00087$). We could work it out in a little more detail, but our conclusion is that the two boxes are not on the same *SIC*.
- If we change the box slightly to $q_1^T = 99.75$ and $q_2^T = 76.9$ the difference declines to -0.00026 ($3.78806 - 3.78832$).
- The point is that we can change the dimensions of the box such that the overall value of $\tilde{F}(q_1^T, q_2^T)$ remains unchanged.

Third Box Boundaries

q_1^1	q_2^1	q_1^2	q_2^2	MRS_1	MRS_2	U_1	U_2	$F(U_1, U_2)$
6.214	10.000	93.536	66.900	0.932	0.932	8.267	81.801	3.25828
7.922	12.500	92.078	62.500	0.951	0.982	10.415	78.858	3.35546
9.699	15.000	90.301	60.000	0.970	1.003	12.599	76.679	3.43663
11.549	17.500	88.451	57.500	0.990	1.026	14.820	74.454	3.50307
13.477	20.000	86.523	55.000	1.011	1.049	17.079	72.181	3.55851
15.489	22.500	84.511	52.500	1.033	1.073	19.379	69.857	3.60531
17.589	25.000	82.411	50.000	1.055	1.099	21.720	67.480	3.64504
19.785	27.500	80.215	47.500	1.079	1.126	24.106	65.048	3.67880
22.081	30.000	77.919	45.000	1.104	1.154	26.539	62.556	3.70734
24.485	32.500	75.515	42.500	1.130	1.185	29.020	60.003	3.73119
27.006	35.000	72.994	40.000	1.157	1.217	31.552	57.385	3.75071
29.652	37.500	70.348	37.500	1.186	1.251	34.138	54.697	3.76611
32.432	40.000	67.568	35.000	1.216	1.287	36.781	51.936	3.77750
35.358	42.500	64.642	32.500	1.248	1.326	39.484	49.098	3.78486
38.439	45.000	61.561	30.000	1.281	1.368	42.251	46.178	3.78806
41.690	47.500	58.310	27.500	1.317	1.414	45.085	43.169	3.78684
45.126	50.000	54.874	25.000	1.354	1.463	47.990	40.068	3.78079
48.760	52.500	51.240	22.500	1.393	1.518	50.971	36.867	3.76928
52.613	55.000	47.387	20.000	1.435	1.580	54.033	33.559	3.75144
56.704	57.500	43.296	17.500	1.479	1.649	57.180	30.136	3.72596
61.056	60.000	38.944	15.000	1.526	1.731	60.420	26.589	3.69091
65.694	62.500	34.306	12.500	1.577	1.830	63.759	22.908	3.64329
70.648	65.000	29.352	10.000	1.630	1.957	67.203	19.080	3.57818
75.952	67.500	24.048	7.500	1.688	2.138	70.762	15.089	3.48666
81.643	70.000	18.357	5.000	1.749	2.448	74.443	10.911	3.34991
87.765	72.500	12.235	2.500	1.816	3.263	78.259	6.482	3.11455

A Discussion of Scitovsky Indifference Curves

- As presented in Figure 18, Each *SIC* curve implies a tangency with the Production Possibilities Frontier. This tangency presents a Welfare Optimal
- Second, assume another *SIC* curve (SIC'') this curve implies another optimal (O'_A).
 - In our original *SIC* curve, we set $\psi = 0.5$. Assume that we change this to $\psi = 0.4$.
- It is obvious, that this new point is not optimal given the original *SIC* curve – apply the original parameter ($\psi = 0.5$) to construct a new *SIC* curve (SIC'), this curve is to the left of the original *SIC* curve.

