

Applied General Equilibrium: A Primer

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A Basic setup

- 1 First, let us work out the output demands (q_1, q_2) based on the Constant Elasticity of Substitution utility function and production function based on the Cobb-Douglas production function

$$\begin{aligned} q_1(p_1, p_2, y) &= \frac{\left(\frac{1-a}{a}\right)^{\frac{1}{r-1}} p_1^{\frac{1}{r-1}} y}{\left(\frac{1-a}{a}\right)^{\frac{1}{r-1}} p_1^{\frac{rr-1}{r}} + p_2^{\frac{r}{r-1}}} \\ q_2(p_1, p_2, y) &= \frac{p_2^{\frac{1}{r-1}} y}{\left(\frac{1-a}{a}\right)^{\frac{1}{r-1}} p_1^{\frac{rr-1}{r}} + p_2^{\frac{r}{r-1}}} \\ x_{11}(p_1, w_1, w_2) &= p_1^{\frac{1}{\alpha_1 + \beta_1}} \left(\frac{\alpha_1}{w_1}\right)^{\frac{1-\beta_1}{1-\alpha_1-\beta_1}} \left(\frac{\beta_1}{w_2}\right)^{\frac{\beta_1}{1-\alpha_1}} \\ x_{21}(p_1, w_1, w_2) &= p_1^{\frac{1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1}{w_1}\right)^{\frac{\alpha_1}{1-\alpha_1-\beta_1}} \left(\frac{\beta_1}{w_2}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\beta_1}} \\ x_{12}(p_2, w_1, w_2) &= p_2^{\frac{1}{\alpha_2 + \beta_2}} \left(\frac{\alpha_2}{w_1}\right)^{\frac{1-\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\beta_2}{w_2}\right)^{\frac{\beta_2}{1-\alpha_2}} \\ x_{22}(p_2, w_1, w_2) &= p_2^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2}{w_1}\right)^{\frac{\alpha_2}{1-\alpha_2-\beta_2}} \left(\frac{\beta_2}{w_2}\right)^{\frac{1-\alpha_2}{1-\alpha_2-\beta_2}} \end{aligned} \tag{1}$$

where $x_{11}(p_1, w_1, w_2)$ is the amount of input 1 used to produce output 1, $x_{21}(p_1, w_1, w_2)$ is the amount of input 2 used to produce output 1, $x_{12}(p_2, w_1, w_2)$ is the amount of input 1 used to produce output 2, and $x_{22}(p_2, w_1, w_2)$ is the amount of input 2 used to produce output 2.

- 2 As a starting point, we compute the initial values given $p_1 = 1.15$, $p_2 = 1.00$, $w_1 = 0.25$, $w_2 = 0.35$, $a = 0.50$, $r = 0.4$, $\alpha_1 = 0.35$, $\beta_1 = 0.40$,

$\alpha_2 = 0.40$, and $\beta_2 = 0.35$. First, we consider what the income to households will be for this solution. To do this, note that we have enough information to work out the input demands for each output/input combination. Hence, path

$$w_1 (x_{11} (p_1, w_1, w_2) + x_{12} (p_2, w_1, w_2)) + w_2 (x_{21} (p_1, w_1, w_2) + x_{22} (p_2, w_1, w_2)) = 4.19241 \quad (2)$$

where the total amount of x_1 becomes 8.2501 and x_2 is 6.0854. Substituting this income into the demand functions yields q_1 of 1.7379 and q_2 of 2.1938. We will use this as the focal point for our initial solution.

- 3 Given the parameterization, we can define the system of excess demand equations

$$\begin{aligned} ED_1 (p_1, p_2, w_1, w_2) &= x_{11} (p_1, w_1, w_2)^\alpha x_{21} (p_1, w_1, w_2)^\beta - q_1 (p_1, p_2, y) \\ ED_2 (p_1, p_2, w_1, w_2) &= x_{12} (p_2, w_1, w_2)^\alpha x_{22} (p_2, w_1, w_2)^\beta - q_2 (p_1, p_2, y) \\ ED_3 (p_1, p_2, w_1, w_2) &= 8.2501 - (x_{11} (p_1, w_1, w_2) + x_{12} (p_2, w_1, w_2)) \\ ED_4 (p_1, p_2, w_1, w_2) &= 6.0854 - (x_{21} (p_1, w_1, w_2) + x_{22} (p_2, w_1, w_2)) \\ ED_t (p_1, p_2, w_1, w_2) &= y - w_1 (x_{11} (p_1, w_1, w_2) + x_{12} (p_2, w_1, w_2)) - \\ &\quad w_2 (x_{21} (p_1, w_1, w_2) + x_{22} (p_2, w_1, w_2)) \end{aligned} \quad (3)$$

Note that ED_5 is not really an excess demand equation, it is an income constraint. I will basically substitute this result back into ED_1 and ED_2 . Given this parameterization, we can compute the initial income as 4.1924, and note that the excess demand for $ED_1 = 0.67388$ and $ED_2 = 0.45903$ - so close.

- B Based on these values, we define the “normalized” excess demand for outputs and inputs.

- 1 Normalized in this case implies that we fix $p_2 = 1$. We have to normalize on one price because the system is subject to linear homogeneity. Note that there is a little “wobble” in this solution $ED_1 = 1.2783$, $ED_2 = -0.0725$, $ED_3 = 0$, $ED_4 = 0$, and $ED_5 = 0.0571$. Basically, our initialization does not guarantee the marginal rate of substitution equals the price ratio.
- 2 So, to solve this problem, consider a set of conditions called the complementary slackness conditions

$$\begin{aligned}
ED_1(p_1, p_2, w_1, w_2) p_1 &= 0, \quad ED_1(p_1, p_2, w_1, w_2), p_1 \geq 0 \\
ED_2(p_1, p_2, w_1, w_2) p_2 &= 0, \quad ED_2(p_1, p_2, w_1, w_2), p_2 \geq 0 \\
ED_3(p_1, p_2, w_1, w_2) w_1 &= 0, \quad ED_3(p_1, p_2, w_1, w_2), w_1 \geq 0 \quad . \\
ED_4(p_1, p_2, w_1, w_2) w_2 &= 0, \quad ED_4(p_1, p_2, w_1, w_2), w_2 \geq 0 \\
ED_5(p_1, p_2, w_1, w_2) y &= 0, y \gg 0
\end{aligned} \tag{4}$$

Implicitly, I will assume that income is greater than 0. The complementary slackness conditions state that for any commodity either the excess demand is equal to zero or the price is equal to zero. These are basically the Walrassian conditions. For numerical purposes, I will simplify these conditions slightly

$$\begin{aligned}
ED_1(p_1, p_2, w_1, w_2)^2 p_1 &= 0, \quad p_1 \geq 0 \\
ED_2(p_1, p_2, w_1, w_2)^2 p_2 &= 0, \quad p_2 \geq 0 \\
ED_3(p_1, p_2, w_1, w_2)^2 w_1 &= 0, \quad w_1 \geq 0 \quad . \\
ED_4(p_1, p_2, w_1, w_2)^2 w_2 &= 0, \quad w_2 \geq 0 \\
ED_5(p_1, p_2, w_1, w_2)^2 &= 0, y \gg 0
\end{aligned} \tag{5}$$

implicitly guaranteeing that the excess demands are minimal at zero. The complementary slackness problem can be solved in a variety of ways, but in this case I propose using Wolfram Mathematica to solve

$$\begin{aligned}
\min_{p_1, w_1, w_2} f &= ED_1(p_1, 1, w_1, w_2)^2 p_1 + ED_2(p_1, 1, w_1, w_2)^2 p_2 + \\
&ED_3(p_1, 1, w_1, w_2)^2 w_1 + ED_4(p_1, 1, w_1, w_2)^2 w_2 + \\
&ED_5(p_1, 1, w_1, w_2)^2 \\
\text{s.t. } &p_1 \geq 0, w_1 \geq 0, w_2 \geq 0
\end{aligned} \tag{6}$$

The optimal solution should be $f \rightarrow 0$. In this case, we are not quite there $f = 0.8374$. The solution is $p_1 = 1.0124$, $w_1 = 0.2384$, $w_2 = 0.3219$, and $y = 3.8390$ (note that by assumption $p_2 = 1$). I will work on improving this solution, but this will allow us to discuss the Applied General Equilibrium approach.

C Next, I would like to consider some alternative policy interventions. There is a basic concept called the “Theory of the Second Best” which was formalized by Lipsey, R.G. & Lancaster K. (1956-1957) *The General Theory of the Second Best* *The Review of Economic Studies* 24(1), 11-32. The basic concept is in a market where there are multiple distortions, removing a single distortion is not necessarily guaranteed to move toward a better welfare position.

- 1 As a starting point, I consider adding a tax on good 1 ($\tau_1 = 0.10$) and a subsidy on input 1 ($\tau_2 = -0.05$). This modifies Equation 6

$$\begin{aligned}
\min_{p_1, w_1, w_2} f = & ED_1(p_1 + \tau_1, 1, w_1 + \tau_2, w_2)^2 p_1 + ED_2(p_1 + \tau_1, 1, w_1 + \tau_2, w_2)^2 p_2 + \\
& ED_3(p_1 + \tau_1, 1, w_1 + \tau_2, w_2)^2 w_1 + ED_4(p_1 + \tau_1, 1, w_1 + \tau_2, w_2)^2 w_2 + \\
& ED_5(p_1 + \tau_1, 1, w_1 + \tau_2, w_2)^2 \\
& \text{s.t. } p_1 \geq 0, w_1 \geq 0, w_2 \geq 0
\end{aligned} \tag{7}$$

As would be expected, the p_1 decreases to 0.9186 and the price of the first input increases $w_2 = 0.2883$, but the other price remain about the same at $w_2 = 0.3235$. Interestingly, income increases to 4.2645.

- 2 Next, consider leaving the tax on q_1 , but removing the subsidy form x_1 . This results in a similar solution $p_1 = 0.9137$, $w_1 = 0.2388$, and $w_2 = 0.2330$, but household income goes down to 3.8537.
3. I will leave it as an open question: (a) Do consumer loose as the prices are distorted? (b) Does taking one distortion off improve the solution? Maybe next semester.

Table 1: Production and Input Use under different Policies

Policy	p_1	w_1	w_2	q_1^*	q_2^*	Income	Tax	Subsidy	Total
$\tau_1 = 0, \tau_2 = 0$	1.0124	0.2384	0.3219	1.8882	1.9274	3.8390	0	0	0
$\tau_1 = 0.10, \tau_2 = -0.05$	0.9186	0.28813	0.3235	2.0805	2.1454	4.2645	0.2080	-0.3389	-0.1308
$\tau_1 = 0.10, \tau_2 = 0$	0.9184	0.2388	0.3231	1.8806	1.9385	3.8537	0.1881	0	0.1881