

Applied General Equilibrium : A Primer

A. Basic setup

1. First, let us work out the output demands (q_1, q_2) based on the Constant Elasticity of Substitution utility function and production function based on the Cobb - Douglas production functions

In[441]:=

```
q1[p1_, p2_, y_] := ((1 - a) / a) ^ (1 / (r - 1)) p1 ^ (1 / (r - 1))
y / (((1 - a) / a) ^ (1 / (1 - r)) p1 ^ (r / (r - 1)) + p2 ^ (r / (r - 1))) /. {a -> 0.50, r -> 0.4};
q2[p1_, p2_, y_] := p2 ^ (1 / (r - 1))
y / (((1 - a) / a) ^ (1 / (r - 1)) p1 ^ (r / (r - 1)) + p2 ^ (r / (r - 1))) /. {a -> 0.50, r -> 0.4};
x11[p1_, w1_, w2_] :=
p1 ^ (1 / (1 - alpha1 - beta1)) (alpha1 / w1) ^ ((1 - beta1) / (1 - alpha1 - beta1))
(beta1 / w2) ^ (beta1 / (1 - alpha1 - beta1)) /. {alpha1 -> 0.35, beta1 -> 0.40};
x21[p1_, w1_, w2_] :=
p1 ^ (1 / (1 - alpha1 - beta1)) (alpha1 / w1) ^ (alpha1 / (1 - alpha1 - beta1))
(beta1 / w2) ^ ((1 - alpha1) / (1 - alpha1 - beta1)) /. {alpha1 -> 0.35, beta1 -> 0.40};
x12[p2_, w1_, w2_] :=
p2 ^ (1 / (1 - alpha2 - beta2)) (alpha2 / w1) ^ ((1 - beta2) / (1 - alpha2 - beta2))
(beta2 / w2) ^ (beta2 / (1 - alpha2 - beta2)) /. {alpha2 -> 0.40, beta2 -> 0.35};
x22[p2_, w1_, w2_] :=
p2 ^ (1 / (1 - alpha2 - beta2)) (alpha2 / w1) ^ (alpha2 / (1 - alpha2 - beta2))
(beta2 / w2) ^ ((1 - alpha2) / (1 - alpha2 - beta2)) /. {alpha2 -> 0.40, beta2 -> 0.35};
```

2. Next, given a set of assumptions about output and input prices, let us develop the original solution

In[447]:=

```
orig = {q1[p1, p2, y], q2[p1, p2, y],
x11[p1, w1, w2] + x12[p2, w1, w2], x21[p1, w1, w2] + x22[p2, w1, w2],
y - (w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2]))} /.
{p1 -> 1.15, p2 -> 1.00, w1 -> 0.25, w2 -> 0.35, y -> 4.19241};
Print[orig];
{1.73793, 2.19379, 8.25008, 6.0854, 1.06583 × 10-6}
```

3. Given these assumptions, compute the initial income and excess demand equations for each output

```
In[449]:=
{w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2]) /.
  {p1 → 1.15, p2 → 1.00, w1 → 0.25, w2 → 0.35},
(x11[p1, w1, w2]^0.35) (x21[p2, w1, w2]^0.40) - q1[p1, p2,
  w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])] /.
  {p1 → 1.15, p2 → 1.00, w1 → 0.25, w2 → 0.35},
(x12[p1, w1, w2]^0.40) (x22[p2, w1, w2]^0.35) - q2[p1, p2,
  w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])] /.
  {p1 → 1.15, p2 → 1.00, w1 → 0.25, w2 → 0.35}}

Out[449]=
{4.19241, 0.673882, 0.459034}
```

B. Based on these values, we can define the "normalized" excess demand for outputs and inputs. Normalized refers to the fact that we use the returns to the factors owned by the household to define income

```
In[450]:=
excessDemandN[p1_, p2_, w1_, w2_] := { (x11[p1, w1, w2]^0.35) (x21[p1, w1, w2]^0.40) -
  q1[p1, p2, w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])],
(x12[p2, w1, w2]^0.40) (x22[p2, w1, w2]^0.35) - q2[p1, p2,
  w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])],
orig[[3]] - x11[p1, w1, w2] - x12[p2, w1, w2],
orig[[4]] - x21[p1, w1, w2] - x22[p2, w1, w2], p1 q1[p1, p2,
  w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])] + p2 q2[
  p1, p2, w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])] -
  w1 (x11[p1, w1, w2] + x21[p1, w1, w2]) - w2 (x12[p2, w1, w2] + x22[p2, w1, w2]) };
```

Hence, the normalized excess demand at our original solution becomes

```
In[451]:=
excessDemandN[1.1500, 1.0000, 0.2500, 0.3500]

Out[451]=
{1.27827, -0.0725398, 4.44089 × 10-16, 0., 0.0570145}
```

Next, we solve for general equilibrium holding the price of the second output to be 1.00. This is a form of a complementary slackness formulation

In[452]:=

```
sol1 =
FindMinimum[{excessDemandN[p1, 1, w1, w2] [[1]]^2 p1 + excessDemandN[p1, 1, w1, w2] [[2]]^2 +
  excessDemandN[p1, 1, w1, w2] [[3]]^2 w1 + excessDemandN[p1, 1, w1, w2] [[4]]^2 w2 +
  excessDemandN[p1, 1, w1, w2] [[5]]^2, p1 ≥ 0, w1 ≥ 0, w2 ≥ 0} ,
  {{p1, 1.15}, {w1, 0.25}, {w2, 0.35}}, AccuracyGoal → 11, PrecisionGoal → 9]
```

Out[452]=

```
{0.83735, {p1 → 1.01238, w1 → 0.238436, w2 → 0.32186}}
```

Given these prices, the excess demands should be close to zero

In[453]:=

```
excessDemandN[p1, p2, w1, w2] /. Flatten[{p2 → 1.00, sol1[[2]]}]
```

Out[453]=

```
{0.626126, 0.645781, 0.196873, 0.123816, -0.096205}
```

Next, we compute the overall income based on returns to factors

In[454]:=

```
y1 = (w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])) /.
  Flatten[{sol1[[2]], p2 → 1.00}]
```

Out[454]=

```
3.83897
```

Finally, we compute the level of output and input demands, and excess income.

In[455]:=

```
rs1ts1 = {p1, p2, w1, w2, q1[p1, p2, y], q2[p1, p2, y],
  x11[p1, w1, w2] + x12[p2, w1, w2], x21[p1, w1, w2] + x22[p2, w1, w2],
  y - (w1 (x11[p1, w1, w2] + x12[p2, w1, w2]) + w2 (x21[p1, w1, w2] + x22[p2, w1, w2])),
  0, 0, 0} /. Flatten[{p2 → 1.00, sol1[[2]], y → y1}]
```

Out[455]=

```
{1.01238, 1., 0.238436, 0.32186, 1.88824, 1.92735, 8.05321, 5.96158, 0., 0, 0, 0}
```

C. Next, consider introducing two distortions into the general equilibrium - a tax on the first output and a subsidy on the first input

```
In[456]:=
sol2 = FindMinimum[{excessDemandN[p1 + tau1, 1, w1 + tau2, w2][[1]]^2 p1 +
  excessDemandN[p1 + tau1, 1, w1 + tau2, w2][[2]]^2 +
  excessDemandN[p1 + tau1, 1, w1 + tau2, w2][[3]]^2 w1 +
  excessDemandN[p1 + tau1, 1, w1 + tau2, w2][[4]]^2 w2 +
  excessDemandN[p1 + tau1, 1, w1 + tau2, w2][[5]]^2, p1 ≥ 0, w1 ≥ 0, w2 ≥ 0} /.
  {tau1 → 0.10, tau2 → -0.05}, {{p1, 1.0123}, {w1, 0.2384}, {w2, 0.3219}},
  AccuracyGoal → 11, PrecisionGoal → 9]
```

```
Out[456]=
{0.797595, {p1 → 0.918616, w1 → 0.288313, w2 → 0.323504}}
```

Again, let us compute the excess demand and income at the new equilibrium

```
In[457]:=
excessDemandN[p1 + tau1, p2, w1 + tau2, w2] /.
  Flatten[{p2 → 1.00, sol2[[2]], tau1 → 0.10, tau2 → -0.05}]
```

```
Out[457]=
{0.659186, 0.615164, 0.155176, 0.11717, -0.092853}
```

Income to households

```
In[458]:=
y2 = (w1 (x11[p1 + tau1, w1 + tau2, w2] + x12[p2, w1 + tau2, w2]) +
  w2 (x21[p1 + tau1, w1 + tau2, w2] + x22[p2, w1 + tau2, w2])) /.
  Flatten[{sol2[[2]], p2 → 1.00, tau1 → 0.10, tau2 → -0.05}]
```

```
Out[458]=
4.26461
```

Quantities demanded and supplied, total input demand, amount taxed on q1 and amount subsidized on x1

```
In[459]:=
rs1ts2 = {p1, p2, w1, w2, q1[p1 + tau1, p2, y],
  q2[p1 + tau1, p2, y], x11[p1 + tau1, w1 + tau2, w2] + x12[p2, w1 + tau2, w2],
  x21[p1 + tau1, w1 + tau2, w2] + x22[p2, w1 + tau2, w2],
  y - (w1 (x11[p1 + tau1, w1 + tau2, w2] + x12[p2, w1 + tau2, w2]) +
    w2 (x21[p1 + tau1, w1 + tau2, w2] + x22[p2, w1 + tau2, w2])),
  q1[p1 + tau1, p2, y] tau1, (x11[p1 + tau1, w1 + tau2, w2] + x21[p2, w1 + tau2, w2]) tau2,
  q1[p1 + tau1, p2, y] tau1 + (x11[p1 + tau1, w1 + tau2, w2] + x21[p2, w1 + tau2, w2]) tau2} /.
  Flatten[{p2 → 1.00, sol2[[2]], y → y2, tau1 → 0.10, tau2 → -0.05}]
```

```
Out[459]=
{0.918616, 1., 0.288313, 0.323504, 2.08046,
  2.14541, 8.0949, 5.96823, 0., 0.208046, -0.338862, -0.130816}
```

D. Next, we consider taking the subsidy off of x2, but leaving the tax on q1

```
In[460]:= sol3 = FindMinimum[{excessDemandN[p1 + tau1, 1, w1 + tau2, w2] [[1]]^2 p1 +
    excessDemandN[p1 + tau1, 1, w1 + tau2, w2] [[2]]^2 +
    excessDemandN[p1 + tau1, 1, w1 + tau2, w2] [[3]]^2 w1 +
    excessDemandN[p1 + tau1, 1, w1 + tau2, w2] [[4]]^2 w2 +
    excessDemandN[p1 + tau1, 1, w1 + tau2, w2] [[5]]^2, p1 ≥ 0, w1 ≥ 0, w2 ≥ 0} /.
    {tau1 → 0.10, tau2 → 0}, {{p1, 1.0123}, {w1, 0.2384}, {w2, 0.3219}},
    AccuracyGoal → 11, PrecisionGoal → 9]
```

```
Out[460]= {0.796152, {p1 → 0.918372, w1 → 0.238837, w2 → 0.323037}}
```

Again, we compute the excess demands at the current solution

```
In[461]:= excessDemandN[p1 + tau1, p2, w1 + tau2, w2] /.
    Flatten[{p2 → 1.00, sol3[[2]], tau1 → 0.10, tau2 → 0}]
```

```
Out[461]= {0.65784, 0.614623, 0.18591, 0.118215, -0.090515}
```

Compute the new income to households

```
In[462]:= y3 = (w1 (x11[p1 + tau1, w1 + tau2, w2] + x12[p2, w1 + tau2, w2]) +
    w2 (x21[p1 + tau1, w1 + tau2, w2] + x22[p2, w1 + tau2, w2])) /.
    Flatten[{sol3[[2]], p2 → 1.00, tau1 → 0.10, tau2 → 0}]
```

```
Out[462]= 3.85365
```

```
In[463]:= rslts3 = {p1, p2, w1, w2, q1[p1 + tau1, p2, y],
    q2[p1 + tau1, p2, y], x11[p1 + tau1, w1 + tau2, w2] + x12[p2, w1 + tau2, w2],
    x21[p1 + tau1, w1 + tau2, w2] + x22[p2, w1 + tau2, w2],
    y - (w1 (x11[p1 + tau1, w1 + tau2, w2] + x12[p2, w1 + tau2, w2]) +
    w2 (x21[p1 + tau1, w1 + tau2, w2] + x22[p2, w1 + tau2, w2])),
    q1[p1 + tau1, p2, y] tau1, (x11[p1 + tau1, w1 + tau2, w2] + x21[p2, w1 + tau2, w2]) tau2,
    q1[p1 + tau1, p2, y] tau1 + (x11[p1 + tau1, w1 + tau2, w2] + x21[p2, w1 + tau2, w2]) tau2} /.
    Flatten[{p2 → 1.00, sol3[[2]], y → y3, tau1 → 0.10, tau2 → 0}]
```

```
Out[463]= {0.918372, 1., 0.238837, 0.323037, 1.88058,
    1.93852, 8.06417, 5.96718, 0., 0.188058, 0., 0.188058}
```

```
In[466]:=
```

```
{rslts1, rslts2, rslts3}
```

```
Out[466]=
```

```
{ {1.01238, 1., 0.238436, 0.32186, 1.88824, 1.92735, 8.05321, 5.96158, 0., 0, 0, 0},  
  {0.918616, 1., 0.288313, 0.323504, 2.08046, 2.14541, 8.0949, 5.96823,  
    0., 0.208046, -0.338862, -0.130816}, {0.918372, 1., 0.238837, 0.323037,  
    1.88058, 1.93852, 8.06417, 5.96718, 0., 0.188058, 0., 0.188058} }
```

```
In[465]:=
```

```
{y1, y2, y3}
```

```
Out[465]=
```

```
{3.83897, 4.26461, 3.85365}
```