

Lecture XX: Capital Market

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Readings

Back to the Consumer's Problem

The Production Problem

Readings

- ▶ Friedman & Friedman Introduction & Chapter 9

Back to the Consumer's Problem

- Consider the consumer's problem where the consumer is choosing between consuming now or later

$$\begin{aligned} \max_{C_t, C_{t+1}} \quad & U(C_t, C_{t+1}) \\ \text{s.t.} \quad & p_t C_t + p_{t+1} C_{t+1} \leq W_0 \end{aligned} \quad (1)$$

Taking the ratio of the first-order conditions for each good

$$\frac{\frac{\partial U(.)}{\partial C_t}}{\frac{\partial U(.)}{\partial C_{t+1}}} = \frac{\lambda p_t}{\lambda p_{t+1}} \Rightarrow \left. \frac{\partial C_{t+1}}{\partial C_t} \right|_{U=U^*} = \frac{p_t}{p_{t+1}}. \quad (2)$$

- Normalizing the price we denote

$$\left. \frac{\partial C_{t+1}}{\partial C_t} \right|_{U=U^*} = \frac{1}{1+r}. \quad (3)$$

Consumer Choice Over Time

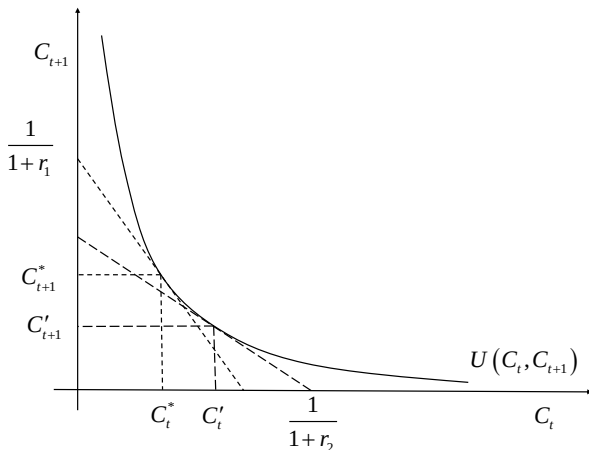


Figure : Consumer Choice Over Time

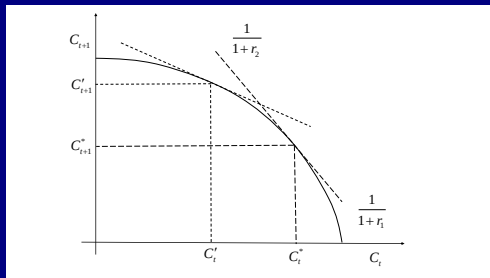
- ▶ In Figure 1, an interest rate of r_1 yields a consumption level of C_t and C_{t+1} .
- ▶ Taking W_0 to be total wealth, the overall consumption becomes

$$W_0 = C_t + (1 + r_1) C_{t+1} \Rightarrow C_{t+1} = \frac{W_0 - C_t}{1 + r_1} \quad (4)$$

- ▶ In this formulation, as the interest rate increases the individual chooses to consume more in the current time period.
- ▶ This result is depicted in Figure 1 when $r_2 \gg r_1$.

The Production Problem

- ▶ Assume that the producer can choose to produce goods today or tomorrow.
- ▶ In my finance book, I consider the scenario where a farmer could either sell potatoes or plant the eyes from the potatoes to produce more next period.
- ▶ The general graph of this tradeoff is depicted in Figure 2.



- Similar to the consumer's problem, we assume that the producer allocates production between this period and next to maximize profit

$$\begin{aligned} \max_{C_t, C_{t+1}} \quad & C_t + (1 + r) C_{t+1} \\ \text{s.t.} \quad & F(C_t, C_{t+1}) = 0 \end{aligned} \quad (5)$$

Forming the Lagrange for Equation 5

$$\frac{1}{1+r} = \frac{\mu \frac{\partial F(.)}{\partial C_t}}{\mu \frac{\partial F(.)}{\partial C_{t+1}}} \Rightarrow \frac{1}{1+r} = \left. \frac{\partial C_{t+1}}{\partial C_t} \right|_{F(.)=0} \quad (6)$$

Robinson Crusoe Economy

- ▶ In the Robinson Crusoe economy, the single economic agent is both the consumer and business firm.
- ▶ The complete model of the consumer's allocation is then

$$\begin{aligned} \max_{C_t, C_{t+1}} & U(C_t, C_{t+1}) \\ \text{s.t.} & F(C_t, C_{t+1}). \end{aligned} \quad (7)$$

utility is maximized where

$$\left. \frac{\partial C_{t+1}}{\partial C_t} \right|_{U=U^*} = \left. \frac{\partial C_{t+1}}{\partial C_t} \right|_{F(.)=0}. \quad (8)$$

Production/Consumption Equilibrium

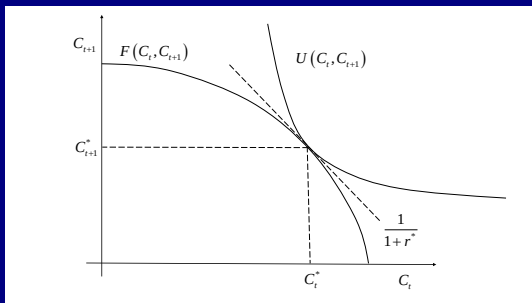


Figure : Production/Consumption Equilibrium

- At this point of equilibrium (assume for a particular consumer), the interest rate that balances consumption and production over the two periods is r^* .

Capital Market

- Next, I want to introduce a simple capital market that will borrow or lend at a new rate r_1 . This result is presented in Figure 4.

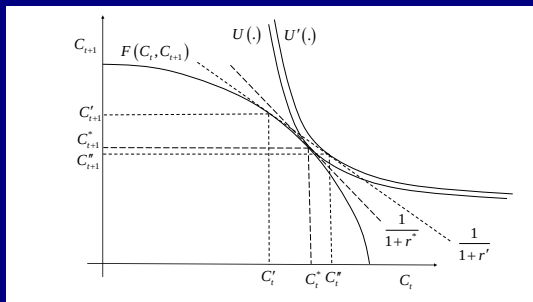


Figure : Capital Market

- ▶ In this market the individual produces (C'_t, C'_{t+1}) and consumes (C''_t, C''_{t+1}) .
- ▶ Essentially, the consumer borrows $B(r) = C''_t(r) - C'_{t+1}(r)$ and pays back $(1 + r)(C'_{t+1}(r) - C''_{t+1}(r))$.
- ▶ This relationship can be used to derive a demand for credit (and a supply of credit) – $B(r)$.

- ▶ Finally, let's assume a credit market

$$B_1(r) + B_2(r) = 0 \Rightarrow B_1(r) = -B_2(r) \quad (9)$$

essentially we derive an excess supply function as in the Back-to-Back diagrams.

- ▶ The capital market equilibrium is then presented in Figure 5.

Borrowing Market

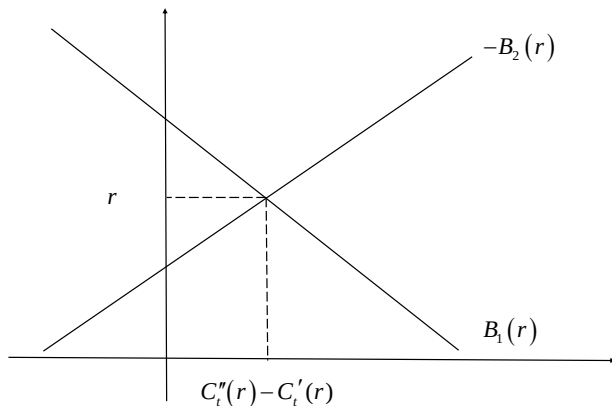


Figure : Borrowing Market