

Lecture XIX: Land and Quasi-Fixed Factors of Production

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Readings

Euler's Theorem and Returns to Factors of Production

Valuing Farmland – Ricardian Rents

Readings

- ▶ Friedman & Friedman Introduction & Chapter 8

Euler's Theorem and Returns to Factors of Production

- A little calculus – a function is homogeneous of degree k if

$$f(\lambda x) = \lambda^k f(x) = f(\lambda x_1, \lambda x_2, \lambda x_3) \quad (1)$$

For a relatively simple example, consider the Cobb-Douglas production function

$$f(\lambda x) = \lambda^{\alpha+\beta} f(x) = (\lambda x_1)^\alpha (\lambda x_2)^\beta. \quad (2)$$

- In Euler's theorem we are interested in the derivative of Equation 1 with respect to λ

$$\begin{aligned}\frac{\partial f(\lambda x)}{\partial \lambda} &= \frac{\partial \lambda^k f(x)}{\partial \lambda} = \frac{\partial f((\lambda x_1), (\lambda x_2), (\lambda x_3))}{\partial \lambda} \\ \Rightarrow k\lambda^{k-1}f(x) &= \frac{\partial f(\lambda x_1, \lambda x_2, \lambda x_3)}{\partial \lambda x_1} \frac{\partial \lambda x_1}{\partial \lambda} + \frac{\partial f(\lambda x_1, \lambda x_2, \lambda x_3)}{\partial \lambda x_2} \frac{\partial \lambda x_2}{\partial \lambda} + \frac{\partial f(\lambda x_1, \lambda x_2, \lambda x_3)}{\partial \lambda x_3} \frac{\partial \lambda x_3}{\partial \lambda}\end{aligned}\quad (3)$$

This expression implies

$$k\lambda^{k-1}f(x) = \frac{\partial f(\lambda x)}{\partial \lambda x_1}x_1 + \frac{\partial f(\lambda x)}{\partial \lambda x_2}x_2 + \frac{\partial f(\lambda x)}{\partial \lambda x_3}x_3. \quad (4)$$

- To finish the specification, we want to focus on the scenario where $k = 1$ called constant returns to scale – if we double all inputs the output doubles. Therefore

$$f(x) = \frac{\partial f(\lambda x)}{\partial \lambda x_1} x_1 + \frac{\partial f(\lambda x)}{\partial \lambda x_2} x_2 + \frac{\partial f(\lambda x)}{\partial \lambda x_3} x_3 \quad (5)$$

- Finally, take the limit as $\lambda \rightarrow 1$ to yield

$$f(x) = y = \frac{\partial f(x)}{\partial x_1} x_1 + \frac{\partial f(x)}{\partial x_2} x_2 + \frac{\partial f(x)}{\partial x_3} x_3 \quad (6)$$

- To this point all we have done is to apply the mathematical implications of homogeneity of degree one. We have not made any economic assumptions. Now, we multiply both sides of Equation 6 by the price of the output

$$p_y y = p_y \frac{\partial f(x)}{\partial x_1} x_1 + p_y \frac{\partial f(x)}{\partial x_2} x_2 + p_y \frac{\partial f(x)}{\partial x_3} x_3. \quad (7)$$

- ▶ Next remember that at the optimum the firm's marginal value product is equal to the input price

$$p_y \frac{\partial f(x)}{\partial x_i} = w_i \quad (8)$$

substituting this result into Equation 8 yields

$$p_y y = w_1 x_1 + w_2 x_2 + w_3 x_3. \quad (9)$$

- ▶ In a constant returns to scale industry the profit is completely absorbed by returns to factors.
- ▶ All factors are paid their marginal product – there are not classical rents as depicted in Figure 1.

Production Function Homogeneous of Degree One

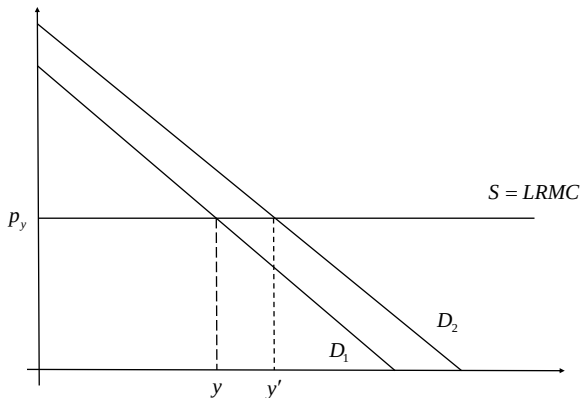


Figure : Production Function Homogeneous of Degree One

Valuing Farmland – Ricardian Rents

- ▶ The valuation of farmland is typically done within the context of Ricardian rent.
- ▶ As a starting point, let us reconsider the formulation Equation 9 such that x_3 is the quantity of farmland used in production and w_3 is the market price for that farmland

$$\begin{aligned} w_3 x_3 &= p_y y - w_1 x_1 - w_2 x_2 \\ w_3 &= \frac{p_y y - w_1 x_1 - w_2 x_2}{x_3} . \end{aligned} \quad (10)$$

- ▶ We could view the price of land as that generated by farmland that is not paid to other resources.
- ▶ The question at least at the time of David Ricardo was that land was abundant – new discoveries had opened land up in the Americas. If land was abundant, why did anyone pay for the services provided by farmland?
- ▶ As a starting point in our discussion of Ricardian rents, consider the square of land depicted in Figure 2.

Land Grid

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15

Figure : Land Grid

- ▶ Each of these squares represents a given quantity of land – say a section (1 square mile or 640 acres).
- ▶ In much of the United States, land is laid out on such a grid. The North-South axis is called a Range while the East-West axis is called a Township. A range/township combination is 36 section - 6 miles north and six miles west.
- ▶ I want to assume that the quality of land varies systematically with the best land being in section 11.
- ▶ I can image a combination of prices such that output price exactly covers the input cost for section 11 (without any allocation to land)

$$p_y y_{11} - w_1 x_{1,11} - w_2 x_{2,11} = 0 \Rightarrow w_{3,11} = 0 \quad (11)$$

in this case no rent is paid to the land in section 11. The farmer makes a normal return on his other inputs.

- Next, assume that the price increases to the point where sections 6 and 12 enter production. Assuming the same yield for each of these sections

$$p'_y y_6 = p'_y y_{12}, \quad p'_y y_6 - w_1 x_{1,6} - w_2 x_{2,y} = 0 \Rightarrow w_{3,6} = w_{3,12} = 0. \quad (12)$$

At this point no rent is paid sections 6 and 12, but the land in section 11 earns a positive rent because

$$\begin{aligned} p'_y y_{11} - w_1 x_{1,12} - w_2 x_{2,12} &\gg p_y y_{11} - w_1 x_{1,12} - w_2 x_{2,12} = 0 \\ &\Rightarrow w_{3,12} \gg 0 \end{aligned} \quad (13)$$

- ▶ In this case Ricardian rents and the valuation of farmland arises because of differences in the productivity of farmland. In a similar specification by Johann Heinrich von Thunen farmland acquires value because it is closer to the point of demand (i.e., a town).
- ▶ In either case, the Ricardian rent is the traditional area of producer surplus.
- ▶ In the current world, it may be important to consider the existence of other quasi-fixed factors of production.

Ricardian Rent

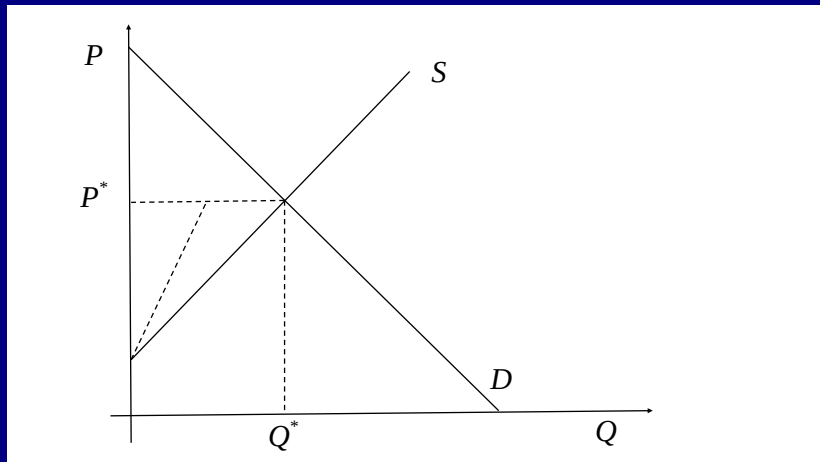


Figure : Ricardian Rent