

# Lecture XVII: Choosing the Firm's Input and Output Level

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Readings

Factor Demand

Supply and Demand of Factors

# Readings

- ▶ Friedman & Friedman Chapter 7

# Factor Demand

- ▶ In Lecture 17, we introduced two firm decision problems – maximize profit and minimize cost for a specific level of output.
- ▶ To develop the demand for factors of production – **Derived Demand** – consider the profit maximization problem

$$\max_{x_1, x_2} \pi = p_y f(x_1, x_2) - w_1 x_1 - w_2 x_2. \quad (1)$$

Taking a small change in each input yields

$$\begin{aligned} \frac{\Delta \pi}{\Delta x_1} &= p_y \frac{\Delta f(x_1, x_2)}{\Delta x_1} - w_1 = 0 \\ \frac{\Delta \pi}{\Delta x_2} &= p_y \frac{\Delta f(x_1, x_2)}{\Delta x_2} - w_2 = 0 \end{aligned} \quad (2)$$

- ▶ In the typical vernacular of economics these relationships state that the Value of Marginal Product equals the price of the input at optimal.
- ▶ Consider the total and marginal product for the quadratic production function presented in Lecture 17 for  $x_1$  holding  $x_2 = 8.01418$  presented in Figure 1

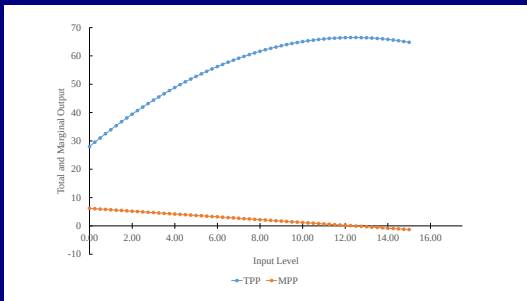


Figure : Total and Marginal Product

- ▶ The total product is

$$f(x_1, x_2) = 5.0x_1 + 6.5x_2 + \frac{1}{2}(-0.50x_1^2 + 0.3x_1x_2 - 0.75x_2^2). \quad (3)$$

Letting  $x_2 = 8.01418$ , the univariate production function becomes

$$f_1(x_1) = 28.007 + 6.20213x_1 - 0.25x_1^2. \quad (4)$$

- ▶ The value of the marginal product is then derived as

$$\frac{\Delta f_1(x_1)}{\Delta x_1} p_y = [6.20213 - 0.5x_1] p_y. \quad (5)$$

- ▶ Next, we equate Equation 5 to  $w_1$  and solve for  $x_1$  yielding

$$x_1(w_1, p_y) = 12.4048 - \frac{2w_1}{p_y}. \quad (6)$$

This is the derived demand for  $x_1$  as a function of its price ( $w_1$ ) and the price of the output ( $p_y$ ). This equilibrium is depicted graphically in Figure 2.

## Derived Demand Equilibrium

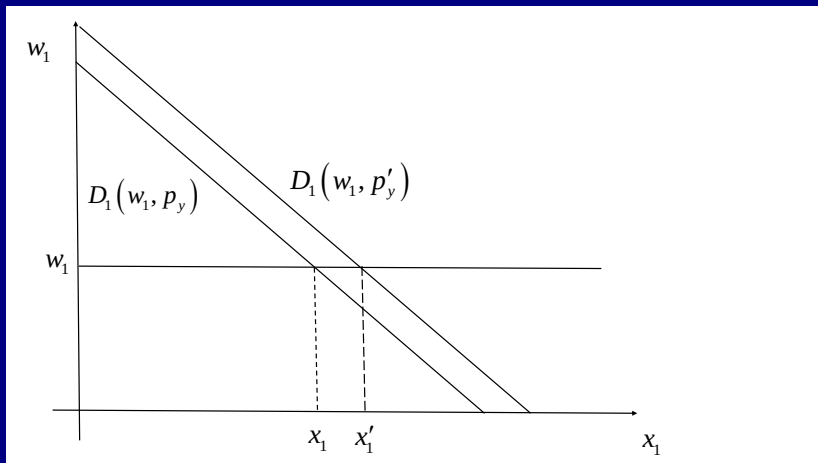


Figure : Derived Demand Equilibrium

- As developed in Lecture 17, the multi-input demand function can be derived from the profit function

$$\max_{x_1, x_2} \pi = p_y \left( a_1 x_1 + a_2 x_2 + \frac{1}{2} (A_{11} x_1^2 + A_{12} x_1 x_2 + A_{22} x_2^2) \right) - w_1 x_1 - w_2 x_2. \quad (7)$$

by taking the derivative with respect to each input and then solving for input levels in terms of input and output prices

$$\begin{aligned} \frac{\Delta \pi}{\Delta x_1} \Rightarrow x_1(p_y, w_1, w_2) &= - \frac{A_{12} a_2 p_y - a_1 A_{22} p_y + A_{22} w_1 - A_{12} w_2}{(A_{12}^2 - A_{11} A_{22}) p_y} \\ \frac{\Delta \pi}{\Delta x_2} \Rightarrow x_2(p_y, w_1, w_2) &= - \frac{A_{12} a_1 p_y - a_2 A_{11} p_y - A_{12} w_1 + A_{11} w_2}{(A_{12}^2 - A_{11} A_{22}) p_y} \end{aligned} \quad (8)$$

The input market for this multiple input scenario is presented in Figure 3

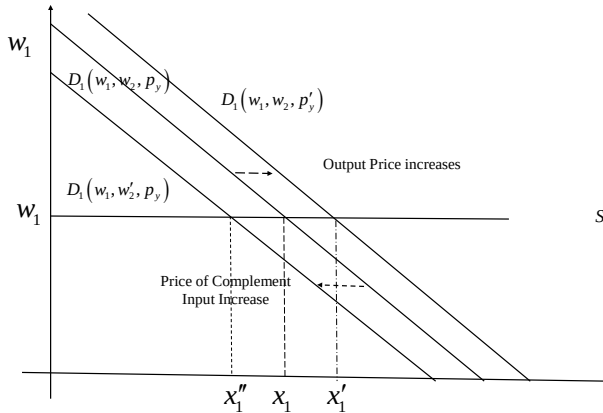


Figure : Derived Demand Equilibrium for more than one Input

# Supply and Demand of Factors

- ▶ A final question deals with the supply of each input.
- ▶ Figures 2 and 3 assume that the supply of each input is perfectly elastic - say farm use of labor.
- ▶ In some input such as agricultural chemicals, the supply curve may not be perfectly elastic as depicted in Figure 4

# Less Elastic Supply of Input

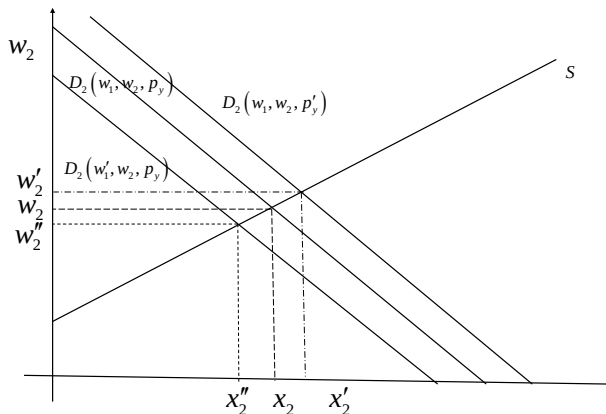


Figure : Less Elastic Supply of Input