

Lecture XV: Stackelberg & Residual Demand

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Readings

Modeling More than One Player – Stackelberg

Residual Demand

Readings

- ▶ Becker Chapter 7 (Lectures 24 – 27)
- ▶ Friedman & Friedman Chapter 4

Modeling More than One Player – Stackelberg

- ▶ Until this point we have considered the extremes – one firm (monopoly) or numerous firms (competition).
- ▶ Consider the case of two firms presented in Figure 1

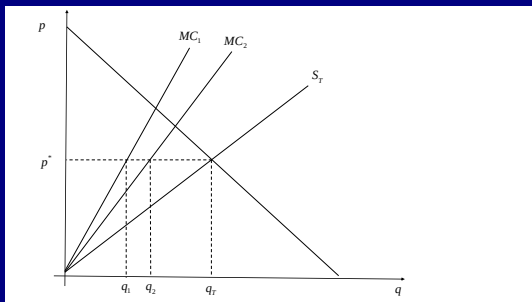


Figure : Competitive Equilibrium

- ▶ The implicit assumption of the model for each firm is

$$\begin{aligned} \max_{q_1} p q_1 - C_1(q_1) \quad \max_{q_2} p q_2 - C_2(q_2) \\ \Rightarrow p = \frac{\partial C_1(q_1)}{\partial q_1} \quad \Rightarrow p = \frac{\partial C_2(q_2)}{\partial q_2} \end{aligned} \quad (1)$$

- ▶ The question is can one of the two producers make more money by conjecturing the other firm's actions.
 - ▶ As a starting place, consider the inverse demand function.
 - ▶ Instead of price determining quantity demanded, the quantity demanded determines the price

$$\begin{aligned} D(p) = q = \alpha_0 + \alpha_1 p \\ D^{-1}(q) = p = \frac{q - \alpha_0}{\alpha_1} \end{aligned} \quad (2)$$

- ▶ Next following the work of Heinrich Freiherr von Stackelberg, I assume that firm 1 conjectures the actions of firm 1

$$\begin{aligned} \max_{q_1} D^{-1}(q_1 + q_2) q_1 - C_1(q_1) & \quad \max_{q_2} p q_2 - C_2(q_2) \\ \Rightarrow p = \frac{\partial D^{-1}(q_1 + q_2) q_1}{\partial q_1} & \quad \Rightarrow p = \frac{\partial C_2(q_2)}{\partial q_2} \end{aligned} \quad (3)$$

- ▶ Suppose that Firm 1 knows or believes that Firm 2's marginal cost function is $MC(q) = \gamma_0 + \gamma_1 q$, then we can derive Firm 2's response function.

- ▶ As a starting point, let us rewrite Equation 2 slightly as

$$p(q_1) = \frac{q_1 + q_2 - \alpha_0}{\alpha_1} \quad (4)$$

- ▶ Substituting this into the implicit marginal cost relationship for Firm 2 yields

$$\begin{aligned} q_2 &= \frac{p - \gamma_0}{\gamma_1} \\ q_2(q_1) &= \frac{\frac{q_1 + q_2(q_1) - \alpha_0}{\alpha_1} - \gamma_0}{\gamma_1} \end{aligned} \quad (5)$$

- ▶ Solving for $q_2(q_1)$

$$q_2(q_1) = \frac{q_1 - \alpha_0 - \gamma_0 \alpha_1}{\gamma_1 - 1}. \quad (6)$$

- Firm 1 then solves the problem

$$\begin{aligned} & \max_{q_1} D^{-1}(q_1 + q_2(q_1)) q_1 - C_1(q_1) \\ \Rightarrow & \frac{\partial D^{-1}(q)}{\partial q} \left(1 + \frac{\partial q_2(q_1)}{\partial q_1} \right) - \frac{\partial C_1(q_1)}{\partial q_1} = 0. \end{aligned} \quad (7)$$

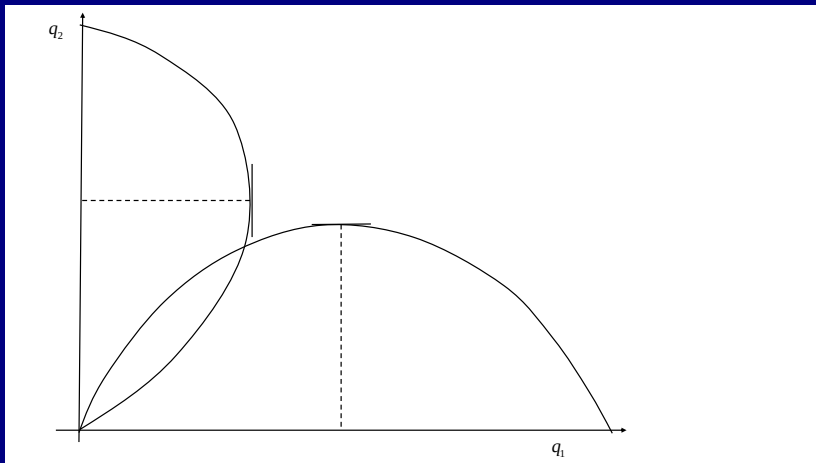


Figure : Stackelberg Equilibrium

Residual Demand

- ▶ A related model is the residual demand model.
- ▶ Returning to Equation 2 suppose that I derived the quantity demanded after the quantity supplied by Firm 2

$$D^{-1}(p) = \frac{q_1 - q_2^*(p) - \alpha_0}{\alpha_1} \quad (8)$$

Residual Demand

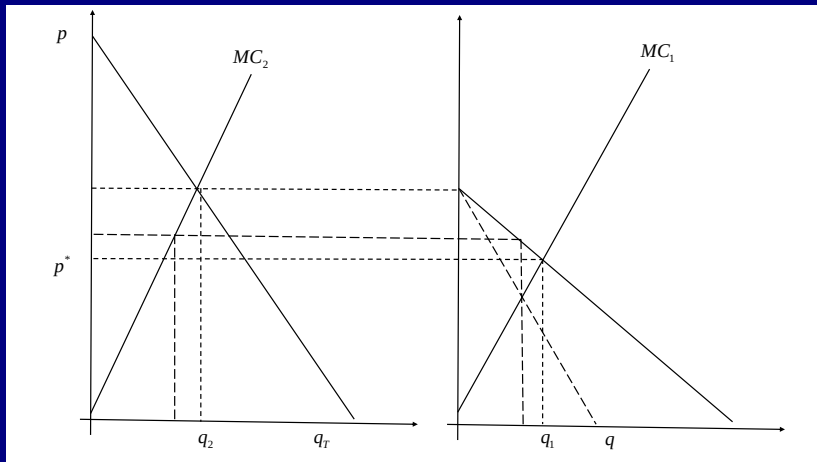


Figure : Residual Demand