

# Lecture VIII: Expected Utility – The Price of Insurance

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Readings

Basic Valuation of Insurance

State-Space Models

# Readings

- ▶ Becker Lecture 15
- ▶ Landsburg Chapters 18 – 19

# Basic Valuation of Insurance

- ▶ I have been trying to avoid mathematical minutiae in this course. However, for the purposes of this discussion, I want to assume a value of the Utility Function

$$U[Y] = -\exp[-\rho Y] = -\exp[-0.00005 Y] \quad (1)$$

- ▶ Figure 1 presents the expected utility of two bets – one bet pays 75,000 with probability 0.40 and 150,000 with probability 0.60 while the second pays 75,000 with probability 0.50 and 150,000 with probability 0.50.

# Expected Utility Graph

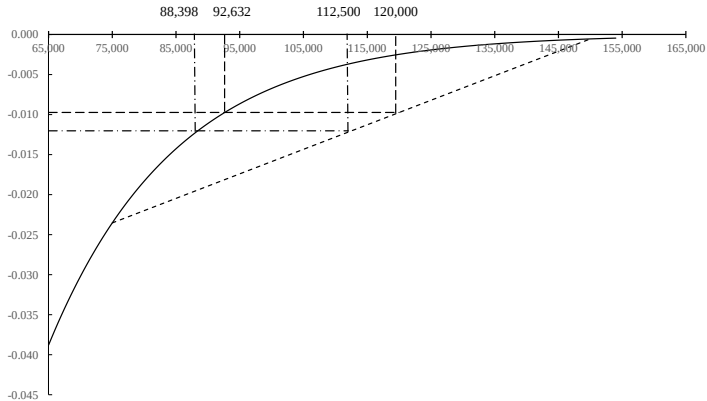


Figure : Expected Utility Graph

- ▶ The expected utility of the first bet is
$$E[U_a] = -0.40 \times 0.023518 - 0.60 \times 0.000553 = -0.00974.$$
- ▶ While the expected utility of the first bet is
$$E[U_b] = -0.50 \times 0.023518 - 0.50 \times 0.00553 = -0.01204.$$

- ▶ Next, I am going to ask what level of income is required to generate a specific level of utility  $U^*$

$$\begin{aligned} -U^* &= \exp[-\rho Y] \\ \ln(-U^*) &= -\rho Y \end{aligned} \tag{2}$$

$$Y^* = -\frac{1}{\rho} \ln(-U^*)$$

- ▶ Thus, the amount of income required to generate the same utility as the first gamble is

$$Y^* = -\frac{1}{0.00005} \ln(-0.00974) = 92,632. \tag{3}$$

- ▶ While the amount of income required to generate the same level of utility as the second gamble is

$$Y^* = -\frac{1}{0.00005} \ln(-0.01204) = 88,398. \tag{4}$$

► Continued

- The risk premium is then the most you would pay to make the first gamble certain is

$$[0.40 \times 75,000 + 0.60 \times 150,000] - 92,632 = 27,367 \quad (5)$$

while the most you would pay to make the second gamble certain is

$$[0.50 \times 75,000 + 0.50 \times 150,000] - 88,398 = 24,102. \quad (6)$$

## State-Space Models

- ▶ There are two ways to develop a state-space model.
  - ▶ My lecture on State-Preference Theory for Financial Decision Making I  
<http://ricardo.ifas.ufl.edu/fin6446.financetheory/Lecture07-1996.pdf> presents the probabilistic formulation.
  - ▶ In this case I want to develop a slightly different problem to highlight the price of insurance.
- ▶ As a starting point, let us adopt the expected utility approach and define the expected utility of a bundle of incomes  $(S_a^0, S_b^0)$  as

$$U(S_a^0, S_b^0) = p_a U(S_a^0) + p_b U(S_b^0) \quad (7)$$

where  $p_a$  is the probability of state  $a$  and  $p_b$  is the probability of state  $b$ .

## State-Preference Formulation

- Suppose this allocation is depicted as point  $P_0$  in Figure 2.

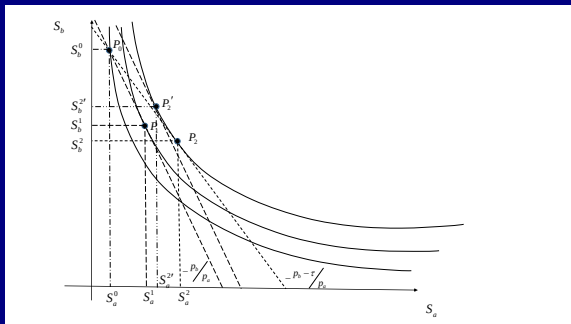


Figure : State-Preference Formulation

- It is clear that point  $P_0$  is not the optimal

$$\Delta U(.) = p_a \frac{\Delta U(S_a)}{\Delta S_a} + p_b \frac{\Delta U(S_b)}{\Delta S_b}$$
$$\Rightarrow -\frac{p_a}{p_b} < \frac{\frac{\Delta U(S_b)}{\Delta S_b}}{\frac{\Delta U(S_a)}{\Delta S_a}}, \quad (8)$$

the marginal rate of substitution is not equal to the price line which in this case is the relative probability line.

- Note that state  $a$  is the high income state – the state where nothing bad happens, while state  $b$  is the low income state – where the consumer gets sick or the car breaks down.

- ▶ If we cannot change the probabilities, we have to move down the probability line by trading a certain amount from one state to another

$$\begin{aligned} S_b^1 &= S_b^0 - \kappa_0 \\ S_a^1 &= S_a^0 - \kappa_0 + \kappa_1. \end{aligned} \tag{9}$$

so the consumer pays an insurance premium of  $\kappa_0$  in both states, but receives a payment of  $\kappa_1$  if the low income state occurs.

- ▶ Graphically, it is possible to define a pair of  $\kappa_0$  and  $\kappa_1$  that leaves the consumer exactly at point  $P_1$ .
- ▶ Next, consider what happens if the government subsidizes this insurance product moving to  $P_2$ . This move yields to overinsurance and an income effect.