

Lecture V: Graphical Derivation of Demand Curves

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Readings

A Brief Rejoinder from Utility Maximization

Graphics of Utility

Development of Demand Theory

Readings

- ▶ Becker Lecture 9 –10
- ▶ Landsburg Chapters 12 – 13

A Brief Rejoinder from Utility Maximization

- ▶ Returning to the basic Lagrangian form from Lecture 4

$$L = U(q_A, q_S) + \lambda(Y - p_A q_A - p_S q_S) \quad (1)$$

suppose we take the change in Equation 1 with respect to the exogenous variables, those variables we cannot control

$$\Delta L = \lambda \Delta Y - q_A \Delta p_A - q_S \Delta p_S. \quad (2)$$

- ▶ Without getting too buried in the calculus, it is important that λ is the marginal utility of income given that the constraint is met

$$\frac{\Delta L}{\Delta Y} = \lambda - q_A \frac{\Delta p_A}{\Delta Y} - q_S \frac{\Delta p_S}{\Delta Y}$$

$$\frac{\Delta [U(q_A, q_S) - \lambda(Y - p_A q_A - p_S q_S)]}{\Delta Y} = \lambda - q_A \frac{\Delta p_A}{\Delta Y} - q_S \frac{\Delta p_S}{\Delta Y} \quad (3)$$

- ▶ Given that $\Delta p_A / \Delta Y = 0$, $\Delta p_S / \Delta Y = 0$, and $Y - p_A q_A - p_S q_S = 0$

$$\frac{\Delta [U(q_A, q_S) - \lambda(Y - p_A q_A - p_S q_S)]}{\Delta Y} = \frac{\Delta U(q_A, q_S)}{\Delta Y} = \lambda. \quad (4)$$

An Alternative Formulation

- ▶ An alternative formulation involves solving for the constraint.
- ▶ Starting with the basic utility maximization problem

$$\Delta U(q_A, q_S) = \frac{\Delta U(.)}{\Delta q_A} \Delta q_A + \frac{\Delta U(.)}{\Delta q_S} \Delta q_S. \quad (5)$$

- ▶ Next, we solve for the changes in q_A and q_S that are consistent with the constraint

$$\begin{aligned} \Delta Y = 0 &= p_A \Delta q_A + p_S \Delta q_S \\ \Rightarrow \Delta q_A &= \frac{p_S}{p_A} \Delta q_S \end{aligned} \quad (6)$$

- Substituting this result back into Equation 5

$$\Delta U(q_A, q_S) = \frac{\Delta U(.)}{\Delta q_A} \left[\frac{p_S}{p_A} \Delta q_S \right] + \frac{\Delta U(.)}{\Delta q_S} \Delta q_S = 0. \quad (7)$$

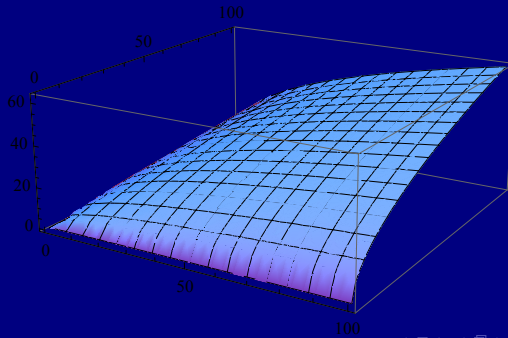
"Solving" Equation 7 for any Δq_S then implies

$$\begin{aligned} \frac{\Delta U(.)}{\Delta q_A} \frac{p_S}{p_A} &= - \frac{\Delta U(.)}{\Delta q_S} \\ \Rightarrow \frac{\frac{\Delta U(.)}{\Delta q_A}}{\frac{\Delta U(.)}{\Delta q_S}} &= - \frac{p_A}{p_S}. \end{aligned} \quad (8)$$

- The result is the same as that developed in Lecture 4 (with the exception of the negative sign).

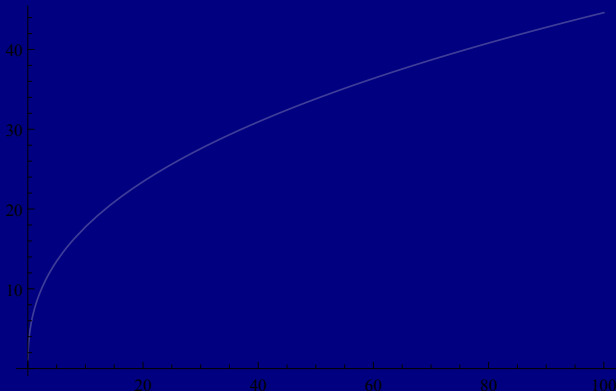
Graphics of Utility

- It is important to remember that the utility function that we have been using is actually a 3-dimensional function as depicted in Figure 1. The value of utility is typically increasing as the quantity of consumption increases.



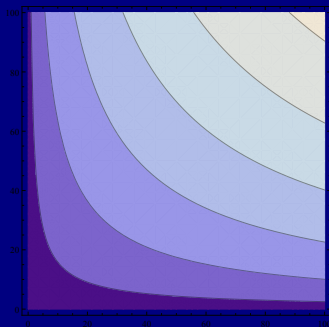
Utility for One Good

- If we take a slice of the utility function – holding $x_2 = 50$ we get a concave surface as depicted in Figure 2.



Isoquants

- ▶ Looking down onto the utility frontier from above is what gives us the standard **isoquant** (the combinations that yield the same level of utility) view of the utility surface depicted in Figure 3.



Development of Demand Theory

- ▶ The Effect of Income - Engel Curves
 - ▶ The first step in developing demand relationships from preferences is to analyze the effect of changes in income the behavior of a rational consumer.
 - ▶ As depicted in Figure 4, increases in income lead to the increased consumption of at least one good.

Changes in Income

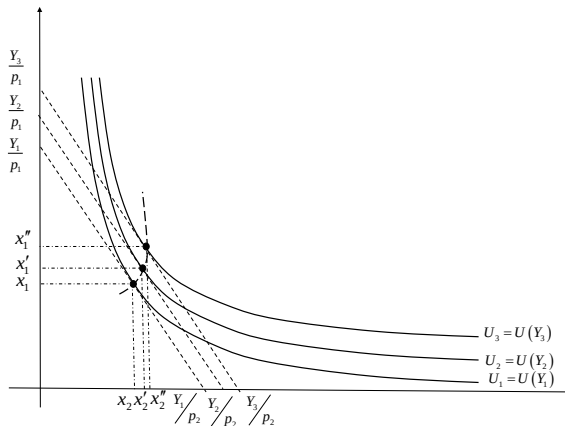


Figure : Changes in Income

- ▶ As depicted in Figure 4, the amount of x_1 demanded by the consumer increases at an increasing rate as income increases.
- ▶ Also as depicted in Figure 4, the amount of x_2 demanded increases at a decreasing rate.
- ▶ In the vernacular of economics good x_1 is a superior good while x_2 is an inferior good.
- ▶ Inferior goods are typically necessities – like food.

- ▶ Food typically obeys Working's Law

$$w_f = \alpha + \beta \ln(Y). \quad (9)$$

The share of food in the consumer's budget declines with an increase in the natural logarithm of income.

Marshallian Demand Curves

- ▶ Marshallian Demand Curves – The Effect of Changes in Price
 - ▶ The second step is to consider a change in the price for one of the goods.
 - ▶ In this case, we assume that the price of x_2 declines causing the budget line to rotate as depicted in Figure 5.

Changes in Price – Marshallian Demand

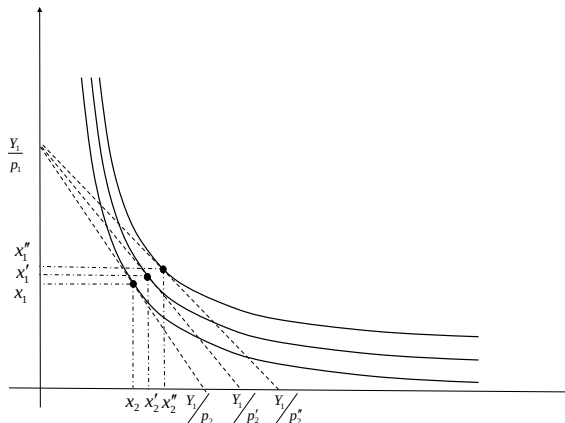


Figure : Changes in Price – Marshallian Demand

Hicksian Demand Curves

- ▶ Hicksian Demand Curves – Adjusting for Implicit Changes in Income
- ▶ The total change in the demand for the commodity can be decomposed into two changes:
- ▶ The shift from p_2 to p'_2 implies an income effect – the point (x_2^*, x_1^*) is on the **Expansion Path** or the Engel Curve in Figure 6.

Change in Price – Hicksian Demand

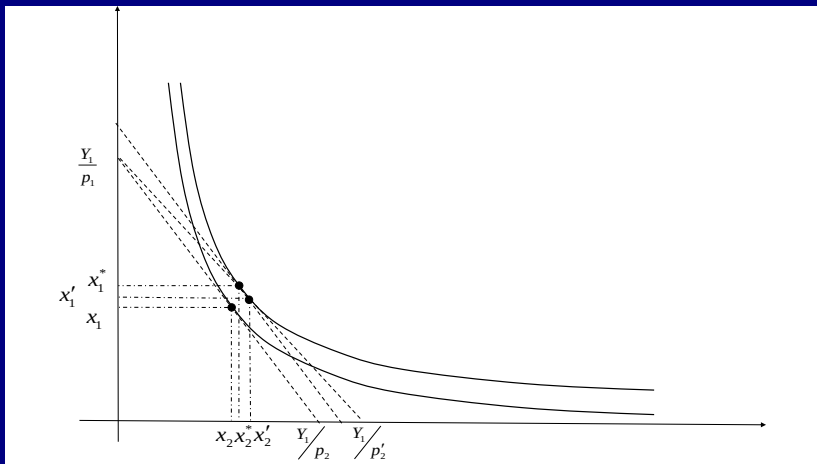


Figure : Change in Price – Hicksian Demand

► Mathematically,

$$\Delta x_2(p_1, p_2, Y) = \frac{dx_2(\cdot)}{dp_1} \Delta p_1 + \frac{dx_2(p_1, p_2, Y)}{dp_2} \Delta p_2 + \frac{dx_2(\cdot)}{dY} \Delta Y. \quad (10)$$

Setting $\Delta p_1 = \Delta p_2 = 0$, dividing through by ΔY , and then multiplying the left-hand side by a form of one $Y/Y \times x_2/x_2$

$$\frac{\Delta x_2(p_1, p_2, Y)}{\Delta Y} = \frac{dx_2(\cdot)}{dY} \frac{Y}{Y} \frac{x_2}{x_2} \quad (11)$$

$$\frac{\Delta x_2(p_1, p_2, Y)}{\Delta Y} = \frac{\frac{dx_2}{x_2}}{\frac{dY}{Y}} \times \frac{x_2}{Y}$$

► Defining

$$\eta_2 = \frac{\frac{dx_2}{dY}}{\frac{x_2}{Y}} \frac{x_2 p_2}{Y} \quad (12)$$

as the income elasticity of demand and $k_2 = x_2 p_2 / Y$ as the income share of good 2

$$\frac{\Delta x_2(p_1, p_2, Y)}{\Delta Y} p_2 = \eta_2 k_2 \quad (13)$$

Demand Curves - Own Price Effect

- The Marshallian and Hicksian Demand functions are presented in Figure 7.

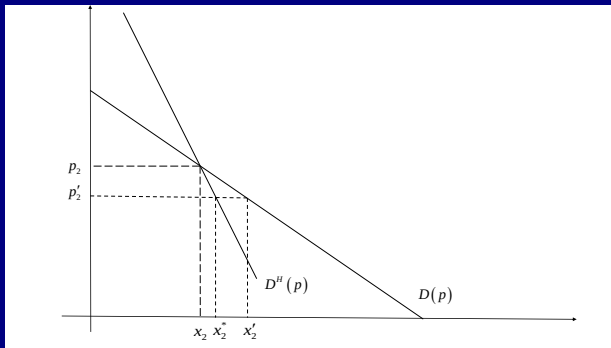


Figure : Effect of Own-Price Change

- ▶ The Hicksian demand function is steeper than Marshallian demand function – the price substitution is higher once the income effect is removed for most goods. Mathematically

$$\epsilon_2 = \epsilon_2^H + \eta_2 k_2 \quad (14)$$

where ϵ_2 is the overall elasticity of demand, ϵ_2^H is the Hicksian elasticity (the pure substitution term), and η_2 is the income elasticity.

- ▶ Equation 14 is typically referred to as the Slutsky decomposition.

Demand Curves – Cross Price Effect

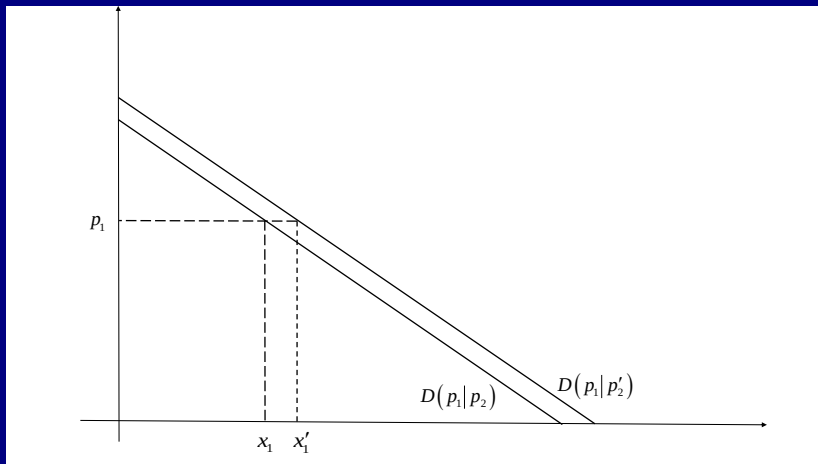


Figure : Effect of Cross-Price Change