

# Lecture IV: Consumer Preferences

Charles B. Moss

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Readings

Consumer Preferences

The Mathematics

# Readings

- ▶ Becker Lecture 8 & 11
- ▶ Landsburg Chapters 10 – 11

# Consumer Preferences

- ▶ In the last lecture we demonstrated that the budget constraint was sufficient to generate downward sloping demand curves and a substitution effect – even through random consumer behavior.
- ▶ To refine the model we introduce random behavior through the concept of consumer preferences.
  - ▶ Historically, some economic theory has assumed the consumer demand as a primitive – a basic model without need for justification.
  - ▶ The basic concept of utility is typically traced to Jerome Bentham.
  - ▶ The great developments of consumer demand based on utility then follow in J.R. Hick's *Value and Capital* and Paul Samuelson's *Foundations of Economic Analysis*.

## Basic Preference of More to Less

- At the most basic level we assume that consumers prefer more to less. This insight implies the right-angle utility structure presented in Figure 1.

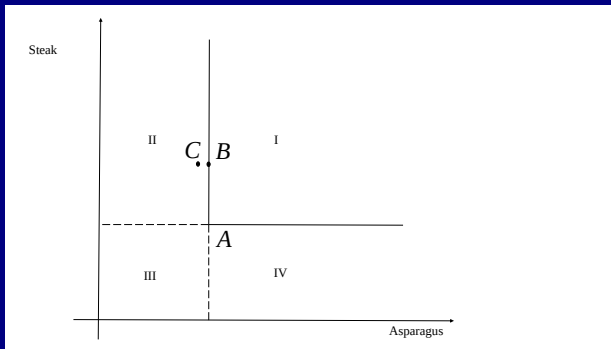


Figure : Basic Preference of More to Less

- ▶ Starting at point  $A$ , it is easy to conjecture that the consumer prefers any combination in Quadrant I – including more of both steak and asparagus.
- ▶ By similar conjecture, point  $A$  is preferred to any point in Quadrant III which contains less of both goods.
- ▶ The real trick involves dividing Quadrants II and IV.
  - ▶ We would suggest that consumers must be at least as satisfied with point  $B$  as point  $A$ .
  - ▶ However, we also suggest that the consumer may also be at least as satisfied with point  $C$  as point  $A$  - there is probably some tradeoff between steak and asparagus.
- ▶ This structure of preferences or utilities are used to develop the rational behavior of consumers – in a sense utility has become our economic primitive.

- ▶ What do we mean by rationality?
  - ▶ Becker – Certainly not that people are necessarily selfish, "economic men" solely concerned with their own well-being.
  - ▶ Becker – Also, rationality should not imply that each household's decisions are necessarily independent of those made by others.
  - ▶ Becker – The essence of this model of rational behavior is contained in just two assumptions: each consumer has an ordered set of preferences, and he chooses the most preferred position available to him.
- ▶ Starting with the concept of transitivity (i.e., if  $A$  is preferred to  $B$  and  $B$  is preferred to  $C$  then  $A$  is preferred to  $C$ ) we can then define the utility function as depicted in Figure 2.

# Defining the Utility Function

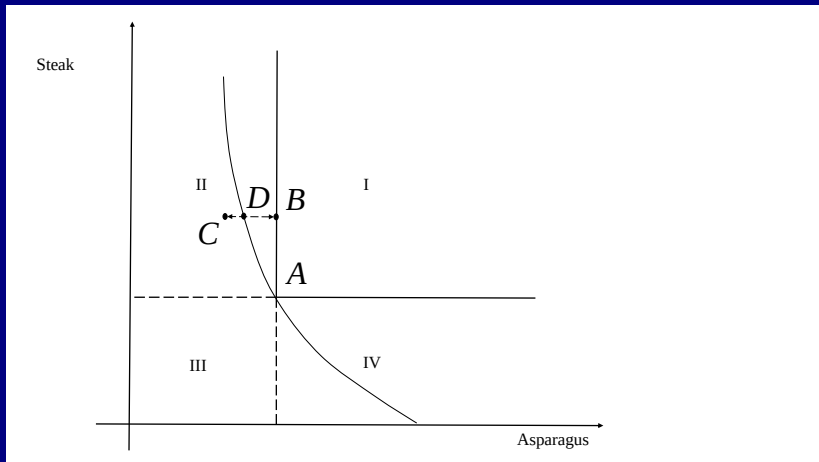


Figure : Defining the Utility Function

- ▶ The concept is that we want to find the set of points such that the consumer is indifferent with point  $A$ .
- ▶ Assume that we can define a point  $C$  such that the consumer prefers  $A$  to  $C$ .
- ▶ Moving from point  $C$  towards point  $B$  we find the point  $D$  such that the consumer is indifferent between  $A$  and  $D$ .
- ▶ Figure 3 presents the family of utility curves based on iterative application of the process above.

# Family of Utility Curves

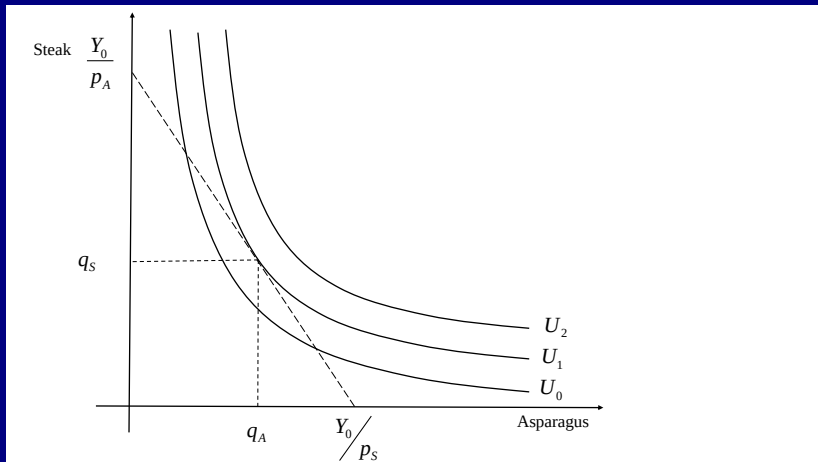


Figure : Family of Utility Curves

- ▶ The curves do not cross.
- ▶ Utility curves lying to the northeast are preferred  
 $U_0 \prec U_1 \prec U_2$ .
- ▶ Typically we assume that the utility functions are convex toward the origin – this assumption yields a unique solution to the consumer's problem.
- ▶ Imposing the budget-line on the utility the point of tangency determines the best that the consumer can do for any level of income.

## A little mathematics

- ▶ As a starting point, consider utility as a numeric function of two variables –  $q_A$  and  $q_S$

$$U(q_A, q_S) = U_i \quad (1)$$

for some number  $U_i$ .

- ▶ The consumer's problem is to maximize the level of utility – generate the highest level of happiness for any level of income. The consumer's problem can be written as

$$\begin{aligned} & \max_{q_A, q_S} U(q_A, q_S) \\ & \text{s.t. } p_A q_A + p_S q_S \leq Y \end{aligned} \quad (2)$$

- ▶ We want to rewrite the basic problem slightly as

$$\max_{q_A, q_S, \lambda} L = U(q_A, q_S) + \lambda(Y - p_A q_A - p_S q_S) \quad (3)$$

- ▶ We have introduced another variable  $\lambda$  which is called the Lagrange multiplier.
- ▶ The easiest way to understand the formulation is to picture firing a cannon around the earth as depicted in Figure 4.

# The Lagrange Multiplier

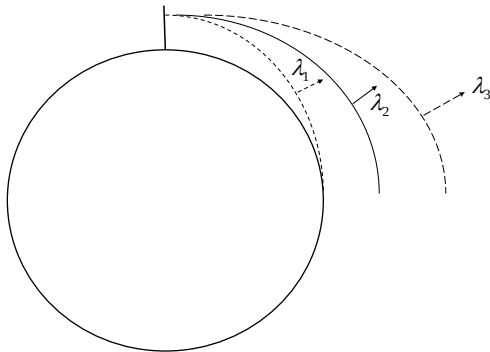


Figure : The Lagrange Multiplier

- ▶ If the cannon is fired with too small a charge the cannon ball falls back to the earth – that level of charge is  $\lambda_1$ .
- ▶ Alternatively, if too much charge is used the cannon ball escapes the gravitational pull –  $\lambda_3$ .
- ▶ However, if the charge is just right the cannon ball achieves a stationary orbit –  $\lambda_2$ .
- ▶ The concept is then to find that level of charge such that the cannon ball follows the constraint – stays in orbit.
- ▶ As depicted in Figure 5 the economic problem is to find the  $\lambda$  that keeps the solution on the income constraint.

# Constrained Consumption

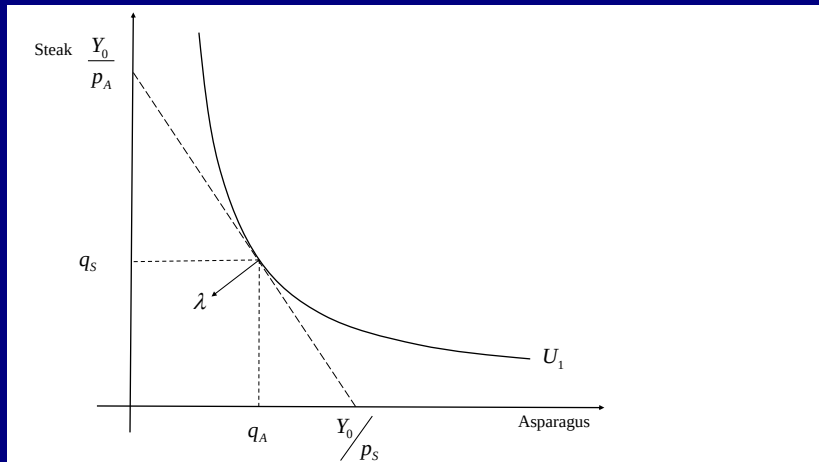


Figure : Constrained Consumption

- We define the maximum level of utility based on the concept that any change would yield a lower level of  $L$  in Equation 3

$$\Delta L = \frac{\Delta U(.)}{\Delta q_A} \Delta q_A + \frac{\Delta U(.)}{\Delta q_S} \Delta q_S - \quad (4)$$

$$\lambda p_A \Delta q_A - \lambda p_S \Delta q_S + \Delta \lambda (Y - p_A q_A - p_S q_S) = 0.$$

- Grouping by the change in choice variables

$$\begin{aligned}\Delta L = & \left[ \frac{\Delta U(.)}{\Delta q_A} - \lambda p_A \right] \Delta q_A + \\ & \left[ \frac{\Delta U(.)}{\Delta q_S} - \lambda p_S \right] \Delta q_S + \\ & [Y - p_A q_A - p_S q_S] \Delta \lambda = 0.\end{aligned}\tag{5}$$

- In order for a point (level of  $q_A$  and  $q_S$ ) to be optimal

$$\frac{\Delta U(.)}{\Delta q_A} - \lambda p_A = 0 \Rightarrow \lambda = \frac{\frac{\Delta U(.)}{\Delta q_A}}{p_A} . \quad (6)$$
$$\frac{\Delta U(.)}{\Delta q_S} - \lambda p_S = 0 \Rightarrow \lambda = \frac{\frac{\Delta U(.)}{\Delta q_S}}{p_S}$$

- Equating the  $\lambda$ s yields

$$\frac{\frac{\Delta U(.)}{\Delta q_A}}{p_A} = \frac{\frac{\Delta U(.)}{\Delta q_S}}{p_S}$$

$$\frac{\frac{\Delta U(.)}{\Delta q_A}}{\frac{\Delta U(.)}{\Delta q_S}} = \frac{p_A}{p_S} . \quad (7)$$

$$\left. \frac{\Delta q_S}{\Delta q_A} \right|_{\Delta U(.)=0} = \frac{p_A}{p_S}$$