

Lecture III: Downward Sloping Demand – Scarcity and Random Behavior

Charles B. Moss

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Readings

Downward Sloping Demand and Random Choice

Price Indices and Demand

Readings

- ▶ Becker Lectures 6 – 7
- ▶ Landsburg Chapters 8 – 9

Downward Sloping Demand and Random Choice

- ▶ In the next lecture we will develop preference or utility theory which allows for the development of consumer preferences based on rational behavior.
- ▶ However, the assumption of rationality is not necessary to support downward sloping demand functions. These can be developed simply based on scarcity.
- ▶ To develop the concept of scarcity based on non-rational behavior we will assume that consumers choose their level of consumption randomly based on the budget constraint.

- ▶ First, let me offer a brief introduction to probability theory and the expectation.
- ▶ First, consider a probability function written as $P[x_i] \geq 0 \forall x_i$.
- ▶ Based on this concept we can think about the expected return of a bet that pays \$ 0 if a coin toss gives a heads and \$ 10 if the coin toss yields a tails

$$E[\text{Bet}] = 0 \times P[x_i = 0] + 10 \times P[x_i = 1] \quad (1)$$

under the classical assumption of "fair coin" yields (i.e., where $P[x_i = 0] = P[x_i = 1] = 0.5$) yields an expected value of \$ 5. In a more general form the return becomes

$$E[Y] = \sum_{i=1}^N P[x_i] x_i = \bar{x} \quad (2)$$

- ▶ If we go from discrete outcomes to continuous outcomes we get

$$E[Y] = \sum_{\Delta x_i \rightarrow 0} P[x_i] x_i = \int_a^b f(x) x dx = \bar{x}. \quad (3)$$

This result is called the Riemann sum and defines the classical integral from calculus.

- ▶ Given that we don't want to go to the integral solution in this course, I will offer a slightly different approach. The integral is basically an area.
- ▶ Consider what we refer to as the uniform distribution function depicted in Figure 1.

Uniform Density Function

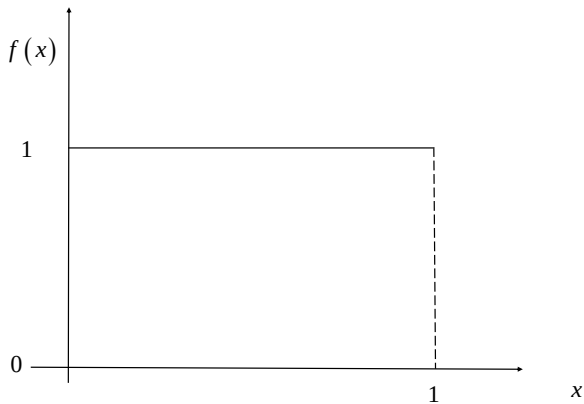


Figure : Uniform Density Function

- ▶ Mathematically, the uniform distribution function is defined as

$$f(x) = 1 \text{ for } x \in [0, 1]. \quad (4)$$

The integral of the density function is simply the area

$$\int_0^1 1dx \Rightarrow \int_a^b 1dx = b - a \Rightarrow 1 - 0 = 1. \quad (5)$$

The expectation of a bet that pays off consistent with the uniform distribution simplifies to the area of a triangle. Specifically, the payoff of the bet becomes

$$\int_a^b 1 \times x dx = \int_a^b x dx \quad (6)$$

which is depicted graphically in Figure 2.

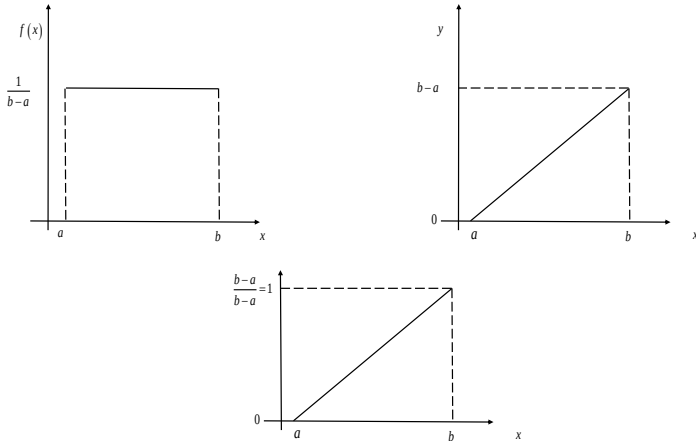


Figure : Expected Value of Uniform Bet

- What is the payoff of the area triangle?

$$\text{Area} = \frac{1}{2} [\text{Width} \times \text{Height}] \quad (7)$$

$$= \frac{1}{2} \times (b - a) \times \frac{b - a}{b - a} = \frac{1}{2} (b - a)$$

- What does this discussion have to do with consumer choice. Recall the consumption choices in Lecture 2 depicted in Figure 3.

Budget Constraints for Beef and Cereal

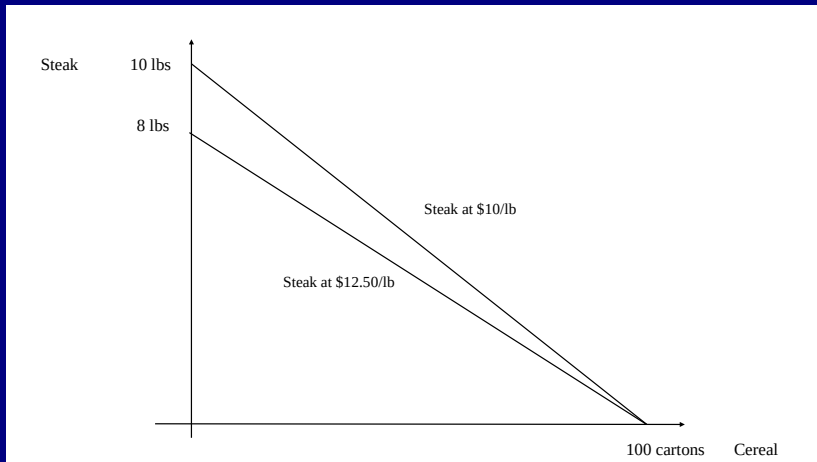


Figure : Budget Constraints for Beef and Cereal

- ▶ What if the consumer randomly chooses the level of consumption according to the first budget constraint (i.e., the constraint based on \$ 10 /lb. steak and \$ 1 / carton cereal)?
- ▶ Following Figure 4, if the consumer randomly selects 8 pounds of beef by the budget constraint implies that they purchase 20 cartons of cereal. Similarly, by randomly selecting 5 pounds of beef, the consumer purchases 50 cartons of cereal.

Randomly Selecting Steak Consumption

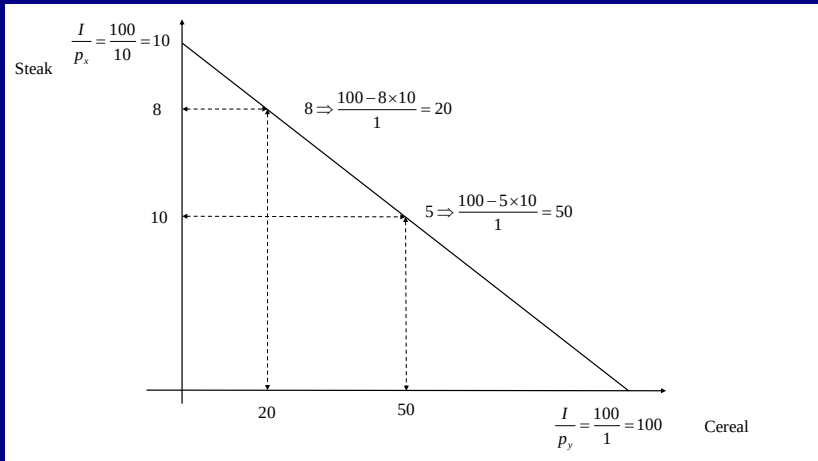


Figure : Randomly Selecting Steak Consumption

- Therefore by either selecting the quantity of beef randomly and then using the rest of the budget on cereal or by randomly choosing the quantity of cereal and then using the rest of the budget on beef the quantity demanded by random consumers are

$$\tilde{x} = \frac{1}{2} \frac{I}{p_x}, \quad \tilde{y} = \frac{1}{2} \frac{I}{p_y} \quad (8)$$

In most ways the results of Equation 8 are sufficient to show that demand curve for each commodity is downward sloping in price and increasing in income.

Price Indices and Demand

- ▶ A little more detail on the demand under scarcity can be derived by returning to the compensation scheme developed in Lecture 2.
- ▶ As depicted in Figure 5 we can derive the amount of income required to make the consumer as well off after the price of beef increases to \$ 12.50 per pound.

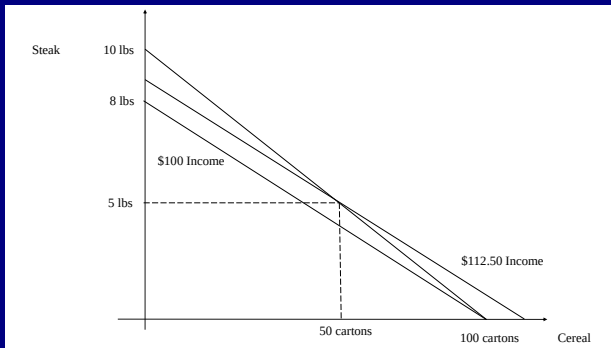


Figure : Increase in Steak Prices

- ▶ Another way to visualize the change is by constructing a Laspeyres index

$$\begin{aligned} P^t &= \sum p_t^t Q_t^0 = \sum \frac{p_t^t}{p_t^0} (p_t^0 Q_t^0) \\ &= 12.50 \times 5 + 1 \times 50 \end{aligned} \tag{9}$$

$$= \frac{12.50}{10.00} (10 \times 5) + \frac{1}{1} (1 \times 50) = 112.50$$

In terms of an inflation index – prices have increased by 12.5 percent (i.e., $112.5/100.0 - 100.0$). Another way to look at it is that we would have to divide the prices by 112.50 to make the same bundle possible after the change in price.

Laspeyres Adjusted Income Constraint

- ▶ The key point is that the consumers may not make the same choice after the prices have been adjusted by the Laspeyres index.
- ▶ Figure 6 presents the constraint adjusted for the Laspeyres index.

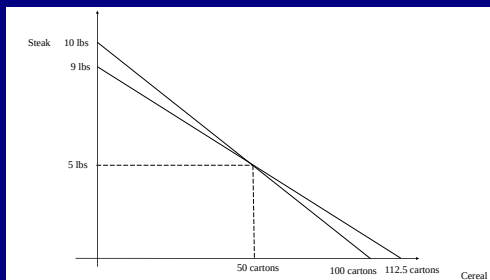


Figure : Laspeyres Adjusted Income Constraint

- ▶ Taking the random choice approach to demand based on this income constraint yields

$$\tilde{x} = \frac{1}{2} \frac{100}{11.111} = 4.5, \tilde{y} = \frac{1}{2} \frac{100}{0.889} = 56.25. \quad (10)$$

Thus, given the compensation for the change in price the demand for steak declines with an increase in the price of steak while the demand for cereal increases as the price of steak increases.

- ▶ Note that change in the price

$$\tilde{p}_x = \frac{12.50}{\frac{112.50}{100.0}} = 11.111, \tilde{p}_y = \frac{1}{\frac{112.5}{100.0}} = 0.889. \quad (11)$$