

Lecture IV: Consumer Preferences

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I. Readings for Next Lecture:

- A. Becker Lecture 8 & 11
- B. Landsburg Chapters 10 – 11

II. Consumer Preferences

- A. In the last lecture we demonstrated that the budget constraint was sufficient to generate downward sloping demand curves and a substitution effect – even through random consumer behavior.
- B. To refine the model we introduce random behavior through the concept of consumer preferences.
 - 1. Historically, some economic theory has assumed the consumer demand as a primitive – a basic model without need for justification.
 - 2. The basic concept of utility is typically traced to Jerome Bentham.
 - 3. The great developments of consumer demand based on utility then follow in J.R. Hick's *Value and Capital* and Paul Samuelson's *Foundations of Economic Analysis*.
- C. At the most basic level we assume that consumers prefer more to less. This insight implies the right-angle utility structure presented in Figure 1.
 - 1. Starting at point *A*, it is easy to conjecture that the consumer prefers any combination in Quadrant I – including more of both steak and asparagus.

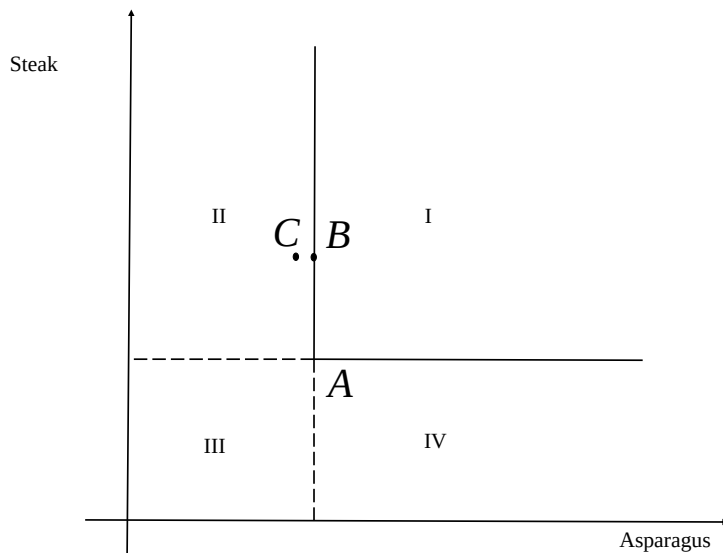


Figure 1: Basic Preference of More to Less

2. By similar conjecture, point *A* is preferred to any point in Quadrant III which contains less of both goods.
3. The real trick involves dividing Quadrants II and IV.
 - a. We would suggest that consumers must be at least as satisfied with point *B* as point *A*.
 - b. However, we also suggest that the consumer may also be at least as satisfied with point *C* as point *A* - there is probably some tradeoff between steak and asparagus.
4. This structure of preferences or utilities are used to develop the rational behavior of consumers – in a sense utility has become our economic primitive.
5. What do we mean by rationality?
 - a. Becker – Certainly not that people are necessarily self-ish, "economic men" solely concerned with their own well-being.
 - b. Becker – Also, rationality should not imply that each household's decisions are necessarily independent of those made by others.

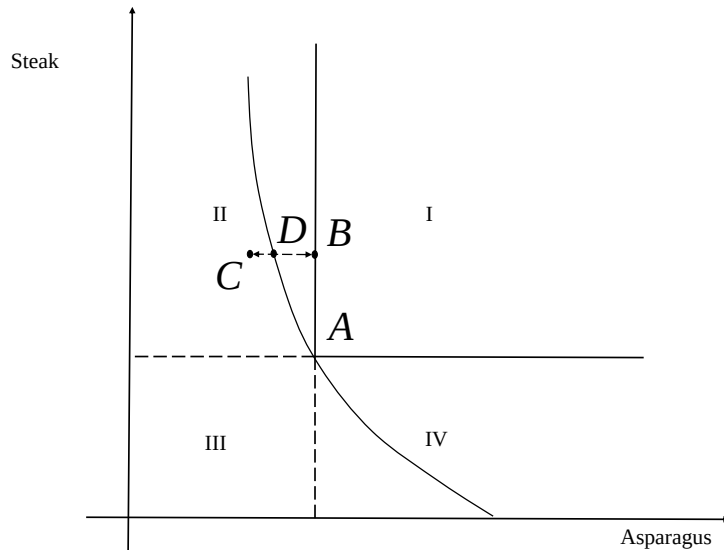


Figure 2: Defining the Utility Function

- c. Becker – The essence of this model of rational behavior is contained in just two assumptions: each consumer has an ordered set of preferences, and he chooses the most preferred position available to him.
- D. Starting with the concept of transitivity (i.e., if A is preferred to B and B is preferred to C then A is preferred to C) we can then define the utility function as depicted in Figure 2.
 1. The concept is that we want to find the set of points such that the consumer is indifferent with point A .
 2. Assume that we can define a point C such that the consumer prefers A to C .
 3. Moving from point C towards point B we find the point D such that the consumer is indifferent between A and D .
- E. Figure 3 presents the family of utility curves based on iterative application of the process above.
 1. The curves do not cross.
 2. Utility curves lying to the northeast are preferred $U_0 \prec U_1 \prec U_2$.

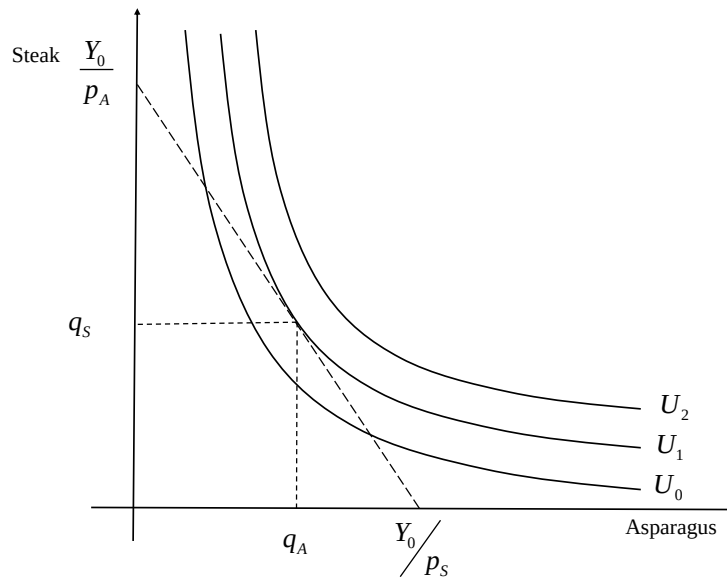


Figure 3: Family of Utility Curves

3. Typically we assume that the utility functions are convex toward the origin – this assumption yields a unique solution to the consumer's problem.
4. Imposing the budget-line on the utility the point of tangency determines the best that the consumer can do for any level of income.

III. A little mathematics

- A. As a starting point, consider utility as a numeric function of two variables – q_A and q_S

$$U(q_A, q_S) = U_i \quad (1)$$

for some number U_i .

- B. The consumer's problem is to maximize the level of utility – generate the highest level of happiness for any level of income. The consumer's problem can be written as

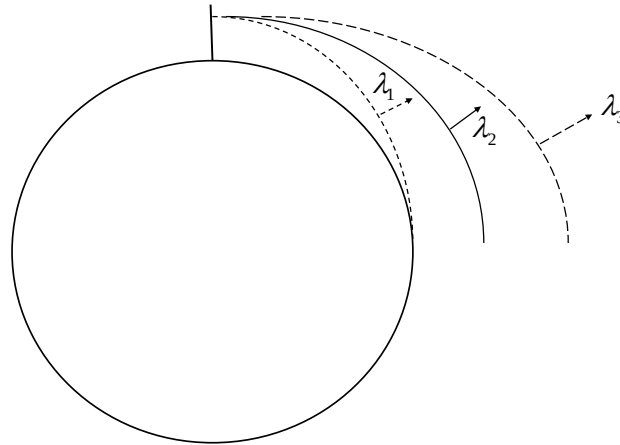


Figure 4: The Lagrange Multiplier

$$\begin{aligned} \max_{q_A, q_S} U(q_A, q_S) \\ \text{s.t. } p_A q_A + p_S q_S \leq Y \end{aligned} \quad (2)$$

1. We want to rewrite the basic problem slightly as

$$\max_{q_A, q_S, \lambda} L = U(q_A, q_S) + \lambda(Y - p_A q_A - p_S q_S) \quad (3)$$

- a. We have introduced another variable λ which is called the Lagrange multiplier.
- b. The easiest way to understand the formulation is to picture firing a cannon around the earth as depicted in Figure 4.
- c. If the cannon is fired with too small a charge the cannon ball falls back to the earth – that level of charge is λ_1 .
- d. Alternatively, if too much charge is used the cannon ball escapes the gravitational pull – λ_3 .
- c. However, if the charge is just right the cannon ball achieves a stationary orbit – λ_2 .

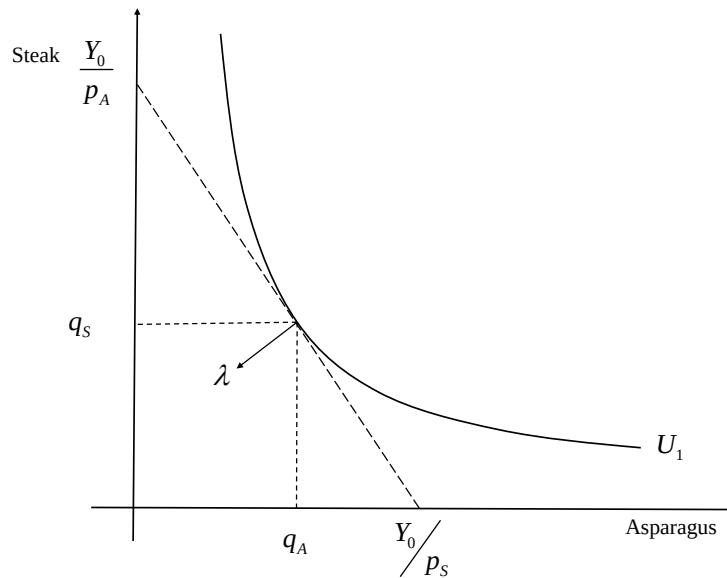


Figure 5: Constrained Consumption

- d. The concept is then to find that level of charge such that the cannon ball follows the constraint – stays in orbit.
- e. As depicted in Figure 5 the economic problem is to find the λ that keeps the solution on the income constraint.
2. We define the maximum level of utility based on the concept that any change would yield a lower level of L in Equation 3

$$\Delta L = \frac{\Delta U(.)}{\Delta q_A} \Delta q_A + \frac{\Delta U(.)}{\Delta q_S} \Delta q_S - \quad (4)$$

$$\lambda p_A \Delta q_A - \lambda p_S \Delta q_S + \Delta \lambda (Y - p_A q_A - p_S q_S) = 0.$$

Grouping by the change in choice variables

$$\begin{aligned}\Delta L = & \left[\frac{\Delta U(.)}{\Delta q_A} - \lambda p_A \right] \Delta q_A + \\ & \left[\frac{\Delta U(.)}{\Delta q_S} - \lambda p_S \right] \Delta q_S +\end{aligned}\tag{5}$$

$$[Y - p_A q_A - p_S q_S] \Delta \lambda = 0.$$

3. In order for a point (level of q_A and q_S) to be optimal

$$\begin{aligned}\frac{\Delta U(.)}{\Delta q_A} - \lambda p_A = 0 & \Rightarrow \lambda = \frac{\frac{\Delta U(.)}{\Delta q_A}}{p_A} \\ \frac{\Delta U(.)}{\Delta q_S} - \lambda p_S = 0 & \Rightarrow \lambda = \frac{\frac{\Delta U(.)}{\Delta q_S}}{p_S}\end{aligned}\tag{6}$$

4. Equating the λ s yields

$$\begin{aligned}\frac{\frac{\Delta U(.)}{\Delta q_A}}{p_A} &= \frac{\frac{\Delta U(.)}{\Delta q_S}}{p_S} \\ \frac{\frac{\Delta U(.)}{\Delta q_A}}{\frac{\Delta U(.)}{\Delta q_S}} &= \frac{p_A}{p_S}\end{aligned}\tag{7}$$

$$\left. \frac{\Delta q_S}{\Delta q_A} \right|_{\Delta U(.)=0} = \frac{p_A}{p_S}$$