

Homework 2: Market Equilibrium and Consumer and Producer Surplus

Charles B. Moss

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1. Following the paper that I will present in Ames Iowa next week. I assume that the cotton price in Oklahoma in 1960 was \$ 30.80 / cwt and the quantity produced was 2172.04 cwt. Assume that the adoption of cotton stripers shifts the supply curve out by 3.83 percent. With a demand elasticity of -0.75 and a supply elasticity of 0.50, what is the new price and quantity?

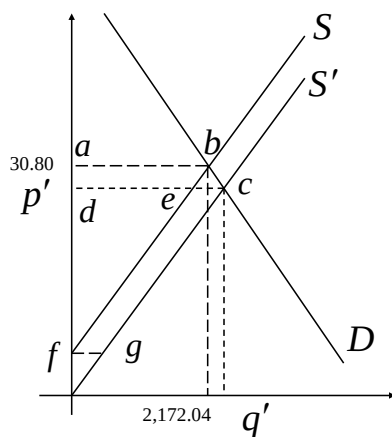


Figure 1: Cotton Shift

- Following the results in Figure 1, I want to construct the linear model of supply

$$Q_S = a_S + b_S p. \quad (1)$$

I know that

$$\epsilon_S = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} \Rightarrow \frac{\Delta Q}{\Delta P} = \epsilon_S \frac{Q}{P} = 0.50 \frac{2,172.04}{30.80} \Rightarrow b_S = 35.26039. \quad (2)$$

To compute the constant in the supply equation

$$\begin{aligned} a_S &= 2,172.04 - 35.26039 \times 30.80 = 1,086.02. \\ \therefore Q_S &= 1,086.02 + 35.26039p \end{aligned} \quad (3)$$

- The solution of the demand curve follows the same general approach

$$\begin{aligned} b_D &= -0.75 \times \frac{2,172.04}{30.80} = -52.8906 \\ a_D &= 2,172.04 + 52.8906 \times 30.80 = 3,801.07. \\ \therefore Q_D &= 3,801.07 - 52.8906p \end{aligned} \quad (4)$$

- It is usually worth your time to check the solution

$$\begin{aligned} Q_S &= 1,086.02 + 35.26039 \times 30.80 = 2,172.04 \\ Q_D &= 3,801.07 - 52.8906 \times 30.80 = 2,172.04 \end{aligned} \quad (5)$$

- Next, I want to shift the supply out by 3.83 percent. I want to shift this supply out so that at the original price the quantity demanded is 3.83 percent higher and the slope of the supply curve remains unchanged. The simplest way to do this is to change the supply to

$$\tilde{a}_S = 2,172.04 \times (1.0383) - 35.26039 \times 30.80 = 1,169.124. \quad (6)$$

Again, check this result

$$Q_S = 1,169.124 + 35.26039 \times 30.80 = 2,255.144 \Leftarrow \frac{2,255.144}{2,172.04} = 1.038261. \quad (7)$$

- To solve for the new price and quantity, I start by equating the new supply with the old demand

$$\begin{aligned} 1,169.124 + 35.26039p &= 3,801.07 - 52.8906 \\ 2,631.946 &= 88.15099p \Rightarrow p = \frac{2,631.946}{88.15099} = 29.85725. \end{aligned} \quad (8)$$

Solving for the new quantity

$$Q_S = 1,169.124 + 35.26039 \times 29.85725 = 2,221.902. \quad (9)$$

2. If we assume a choke price of \$ 0 / cwt (at f and g) what are the areas representing the change in consumer and producer surplus in Figure 1.
 - The additional consumer surplus is $abcd$.
 - The additional producer surplus is $ecgf - abed$.
3. Estimate the (**change in**) magnitudes of consumer surplus and producer surplus.
 - The change in consumer surplus is relatively straightforward

$$\begin{aligned} \Delta CS &= 2,172.04 \times (30.80 - 29.85725) + \\ &\frac{1}{2} (2,221.902 - 2,172.04) \times (30.80 - 29.85725) = 2,071.194. \end{aligned} \quad (10)$$

- We then compute the change in producer surplus in two steps. The first step is the area $abed$ – the loss to producers because of the price decline. First, we compute the quantity at e

$$e \Rightarrow Q_S = 1,086.02 + 35.26039 \times 29.85725 = 2,138.798. \quad (11)$$

The first change in producer surplus $\Delta_1 PS$ ($= abed$) is then

$$\Delta_1 PS = 2,138.798 \times (30.80 - 29.85725) + \frac{1}{2} (2,172.04 - 2,138.798) \times (30.80 - 29.85725) = 2,032.021. \quad (12)$$

- The second part of the change to producer surplus is $ecgf$. Note that we are going to assume a choke point at $p = 0$. This implies that $f = 1,086.02$ and $g = 1,169.124$ – these are the constants in the original supply function and the shifted supply function respectively.

- * One way to estimate this area is $dcg0$ (i.e., where 0 is the quantity equal 0) less the area $def0$

$$dcg0 = 1,169.124 \times (29.85725 - 0) + \frac{1}{2} (2,221.902 - 1,169.124) \times (29.85725 - 0) = 50,623.36 \quad (13)$$

The area $def0$ can be computed as

$$def0 = 1,086.02 \times (29.85725 - 0) + \frac{1}{2} (2,172.04 - 1,086.02) \times (29.85725 - 0) = 48638.35 \quad (14)$$

- * So the numbers I presented in class are slightly off. The second change in producer surplus is

$$\Delta_2 PS = 50,623.36 - 48,638.35 = 1,985.006. \quad (15)$$

The overall change in producer surplus is

$$\Delta PS = -2,032.014 + 1,985.006 = -47.0076. \quad (16)$$