

Lecture IV. Derivation of the Cross Country Florida Model

In developing the cross country Florida model, we are interested in quantifying the budget share

$$w_{ic} = \alpha_i + \beta_i \log Q_c + (w_{ic} - \tilde{w}_{ic}) + \epsilon_{ic} \quad (1)$$

where $w_{ic} - \tilde{w}_{ic}$ adjusts for the move from the average price vector $\bar{p}_i$ to the country specific price vector p_{ic} . We can think of $w_{ic} - \tilde{w}_{ic}$ as dw_i . Remembering that

$$dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log E) \quad (2)$$

we can write

$$dw_i = w_i [d(\log p_i - d(\log P)) + w_i d(\log q_i) - w_i [d(\log E) - d(\log P)]] \quad (3)$$

by adding $-w_i d(\log P) + w_i d(\log P) = 0$ where $d(\log P)$ is the Divisia price index. Noting that

$$[d(\log E) - d(\log P)] = d(\log Q_c)$$

which is zero since the quantity is constant, we have

$$dw_i = w_i [d(\log p_i - d(\log P)) + w_i d(\log q_i)]. \quad (3)$$

Further, we know that

$$w_i d(\log q_i) = \theta_i d(\log Q) + \phi \theta_i \left(d(\log p_i) - \sum_{j=1}^n \theta_j d(\log p_j) \right) \quad (4)$$

under preference independence with $d(\log Q) = 0$. Substituting equation (4) into equation (3) yields

$$dw_i = w_i [d(\log p_i - d(\log P)) + \phi \theta_i \left(d(\log p_i) - \sum_{j=1}^n \theta_j d(\log p_j) \right)].$$

Further substituting for the Divisia price index yields

$$dw_i = w_i \left(d(\log p_i) - \sum_{j=1}^n w_j d(\log p_j) \right) + \phi \theta_i \left(d(\log p_i) - \sum_{j=1}^n \theta_j d(\log p_j) \right). \quad (5)$$

Derivation of the cross country model is then directly based on equation (5). First, $w_i = \alpha_i + \beta_i q_c$ and

$$\theta_i = \frac{dE_i}{dE} = \alpha_i + \beta_i (1 + \log E) = \alpha_i + \beta_i q_c^*. \quad (6)$$

The log-change parameters are then parameterized with respect to geometric mean prices yielding

$$d(\log p_j) = \log \left[\frac{p_{jc}}{\bar{p}_i} \right] \quad (7)$$

and w_i is taken as w_{ic} . Equation (5) then becomes

$$(w_{ic} - \tilde{w}_{ic}) = (\alpha_i + \beta_i q_c) \left(\log \left[\frac{p_{ic}}{p_i} \right] - \sum_{j=1}^n (\alpha_j + \beta_j q_c) \log \left[\frac{p_{jc}}{p_j} \right] \right) + \phi(\alpha_i + \beta_i q_c^*) \left(\log \left[\frac{p_{ic}}{p_i} \right] - \sum_{j=1}^n (\alpha_j + \beta_j q_c^*) \log \left[\frac{p_{jc}}{p_j} \right] \right). \quad (8)$$

Substituting equation (8) into equation (1) with $d(\log Q_c) = q_c$ yields

$$w_{ic} = (\alpha_i + \beta_i q_c) + (\alpha_i + \beta_i q_c) \left(\log \left[\frac{p_{ic}}{p_i} \right] - \sum_{j=1}^n (\alpha_j + \beta_j q_c) \log \left[\frac{p_{jc}}{p_j} \right] \right) + \phi(\alpha_i + \beta_i q_c^*) \left(\log \left[\frac{p_{ic}}{p_i} \right] - \sum_{j=1}^n (\alpha_j + \beta_j q_c^*) \log \left[\frac{p_{jc}}{p_j} \right] \right).$$

which is the cross country model.