

State Contingent Distance Functions: Lecture XXXXXII

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- 1 Risk Aversion and Valuing States
- 2 Duality and Benefit and Distance Functions

Risk Aversion and Valuing States

- In general terms Chambers and Quiggin define a value mapping function that aggregates the value of outcomes in each state. This general mapping function is denoted

$$W : Y^S \rightarrow R \quad (1)$$

- Under risk neutrality, this mapping function simply becomes the expected value

$$W(y) = \sum_{s \in \Omega} \pi_s y_s \quad (2)$$

- Similar under expected utility

$$W(y) = \sum_{s \in \Omega} \pi_s u(y_s) \quad (3)$$

- Backing up slightly, to define this value function, we use income as the measure of well-being resulting from a given state of nature. In this case income y_s is defined as

$$y_s = r_s - g(x) \quad (4)$$

where r_s is defined as the revenue in a particular state

$$r_s(z_s) = \sum_m p_{ms} z_{ms} \quad (5)$$

and there are m outputs.

- Note that output levels, output prices, and input prices may vary across states of nature.

Duality and Benefit and Distance Functions

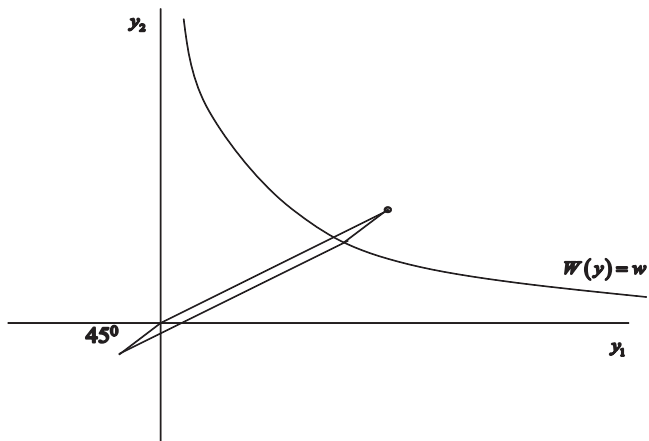
- Following the distance function approach from duality, Chambers and Quiggin develop the benefit function based on general value mapping

$$B(w, y) = \max \{ \beta \in R : W(y - \beta g) \geq w \} \quad (6)$$

if $W(y - \beta g) \geq w$ for some β . For example, assuming risk neutrality

$$W(y) = \sum_{s \in \Omega} \pi_s (y_s - \beta g) \quad (7)$$

Figure : Benefit Function

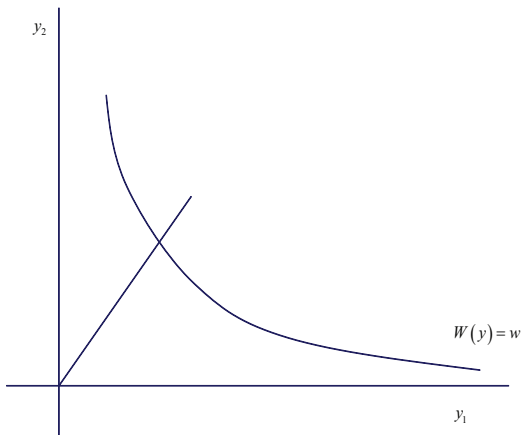


- The point above the output frontier is the observed benefit (remember that the axes are basically incomes in each state).
- The benefit function is then the vertical shift down to the level set (a fixed value of the valuation function).
 - $B(w, y)$ is nonincreasing in w and nondecreasing in y .
 - $B(w, y + \alpha I) = B(w, y) + \alpha$, $\alpha \in R$.
 - $B(w, y) \geq 0 \Leftrightarrow W(y) \geq w$ and $B(w, y) = 0 \Leftrightarrow W(y) = w$.
- A second formulation is the Malmquist distance function D

$$D(y, w) = \sup \left\{ \lambda > 0; W\left(\frac{y}{\lambda}\right) \geq w \right\} \quad (8)$$

Which is the flip side of the distance function discussed above.

Figure : Malmquist Distance Function



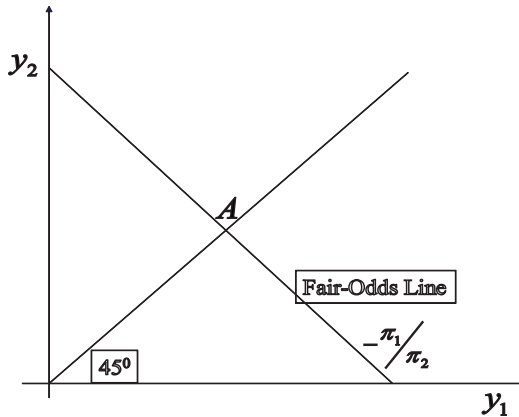
- The certainty equivalent is the smallest certain income that leaves the individual indifferent to the risky gamble. This can be defined based on the state-contingent model as

$$e(y) = \inf \{ c \in R : W(c1) \geq W(y) \} \quad (9)$$

- This function can be represented as a combination of the Malmquist distance function and the benefit function

$$e(y) = \frac{1}{D(1, W(y))} = -B(W(y), 0) \quad (10)$$

Figure : Fair-Odds Line



- In the 2 dimensional state-space graph, the 45^0 line is essentially a risk-free asset. On this line the income is the same for each state.
- Second, all returns on the Fair-odds line have the same expected value.
- Only at the point where the Fair-odds line crosses the 45^0 line are the payoffs risk-free.
- As a result, the decision maker is risk averse with respect to the probability vector π if

$$W(\bar{y}1) \geq W(y) \forall y \quad (11)$$

Indifference implies a convex set of preferences around point . Put differently, the payoffs must be better than fair to make an individual indifferent between the risk-less payoff.

Figure : Distance Function and Fair-Odds Line

