

State Contingent Production Model: Lecture XXXXI

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State Contingent Stochastic Production Functions

- Chambers, Robert G. and John Quiggin. 2002. *Uncertainty, Production, Choice and Agency: The State-Contingent Approach* New York: Cambridge University Press. (Chapters 1 and 2).
- Historically most applications of risk to production theory have been based on a stochastic production function.
 - In its simplest format the stochastic production function is simply a production function with a random error term added to it

$$y = f(x) = x_1^\alpha x_2^\beta + \epsilon \quad (1)$$

Under this formulation the risk (dispersion) created by the error term did not affect the input decisions.

- Continued

- In a slightly more complex scenario, the simple formulation was expanded to allow the risk to be a function of one or both inputs

$$y = \tilde{f}(x) = x_1^\alpha x_2^\beta + \epsilon(x_1, x_2) \quad (2)$$

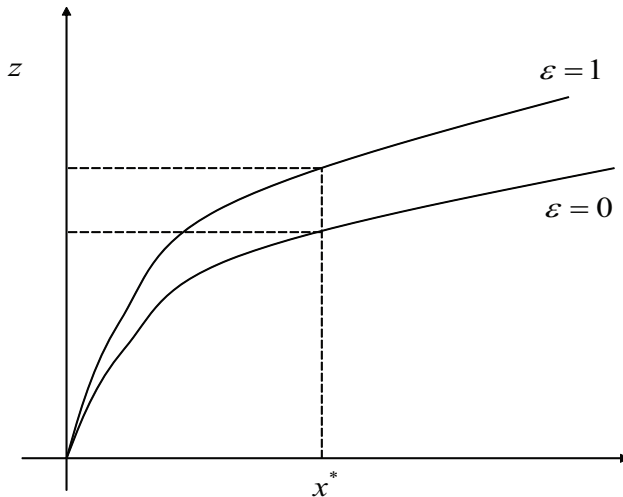
- Even with the expansion of the error function to include relative effects of each input, the assumptions used to parameterize the model imply that the ratio of input use did not change.

- Adapting this formulation slightly, Chambers and Quiggin depict this original production function as dependent on a set of deterministic inputs and a stochastic input

$$z = f(x, \epsilon) \quad (3)$$

- Simplifying this representation, we assume that the random variable is a Bernoulli variable that can obtain to values $\epsilon = \{0, 1\}$. The production function for this representation then depicted in Figure 1.

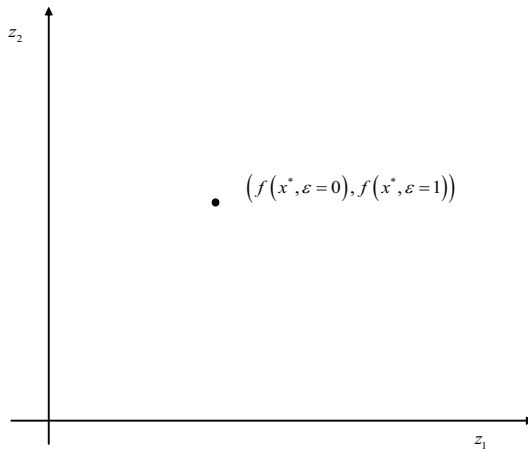
Figure : Stochastic Frontier Production Function



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- The random variable has an effect that is largely independent of input choice, or the introduction of the random variable simply shifts the frontier upward.
- This treatment of inputs is similar to the discussion of disembodied technological change. A disembodied technological change is a change that does not introduce a factor bias or input effect.
- Instead of this approach, Chambers and Quiggin map (or depict) the interaction between input use and random event in a different space (state-contingent output space). In this space, a single production choice implies two different state-space outputs 2.

Figure : Production Possibility Frontier Point



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- These choices are called acts (a) that have different consequences (y) in different states (s).
 - In general, the set of possible states is defined as Ω , the set of possible consequences is defined as $Y \subseteq R^M$, and actions are a mapping from the set of possible states to the set of consequences

$$A : \Omega \rightarrow R^M \quad (4)$$

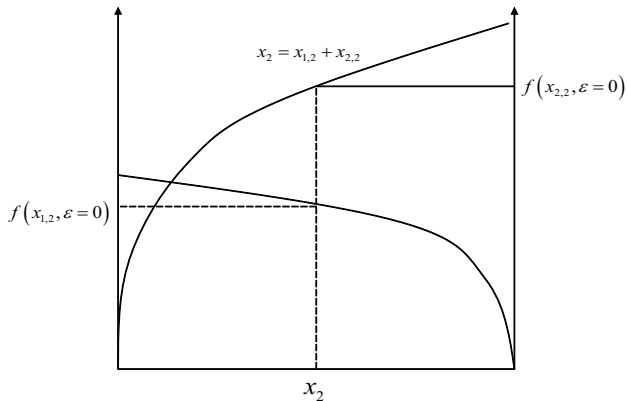
- Each action defines a set state contingent vector of outcomes

$$y(A) \in R^{M \times S} \quad (5)$$

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- It is important to note that the decision maker chooses the action based on its potential consequences for each state.
- Finally, it is important to note that the amount of input applied is dependent on the output state.
 - In the case of the Bernoulli variable above, assume that $\epsilon = 0$ denotes a drought while $\epsilon = 1$ denotes adequate rainfall.
 - In normal years, much of Midwestern agriculture removes excess water from fields using a system of tiles buried under the ground.
 - However, in drought conditions expenditures on drainage may be replaced by expenditures on irrigation.
- Thus, fixing the total amount of money spent on drainage and irrigation, the marginal physical product for each allocation can be represented as depicted in Figure 3.

Figure : Input Tradeoff



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- Reallocating the between x_1 and x_2 yields a smooth tradeoff between the states of nature as depicted in Figure 4.

Figure : State Dependent Frontier

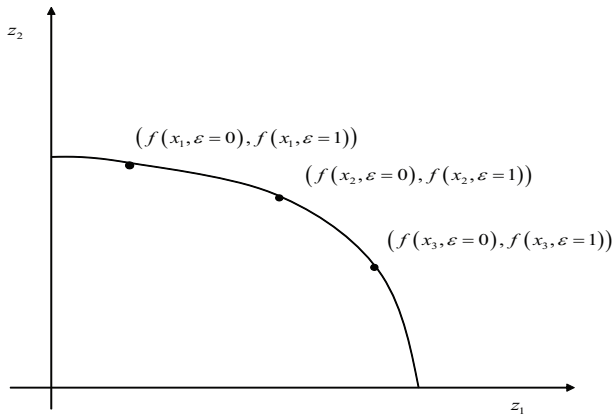
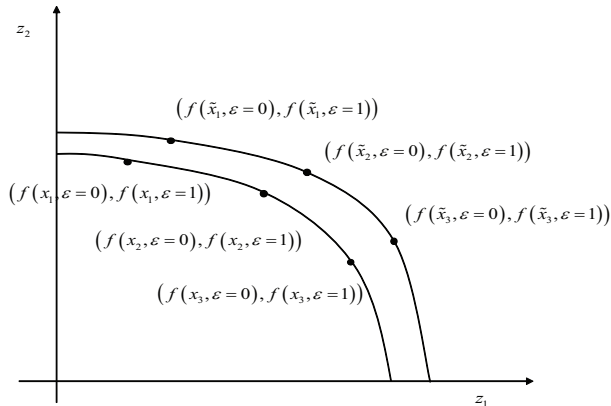


Figure : Shift in State Dependent Frontier



State Production Set and Input Requirement Set

- Based on this state-preference structure, Chambers and Quiggin develop a system similar to that of deterministic production analysis.
 - The *State-contingent output correspondence* is the set of state-contingent outputs that can be generated from a given set of inputs

$$Z(x) = \{ z \in R^{M \times X} : x \in R^N \text{ can produce } z \} \quad (6)$$

- The *Input correspondence* is the set of inputs that can generate a possible set of states

$$X(z) = \{ x \in R^N : x \text{ can produce } z \in R^{M \times S} \} \quad (7)$$

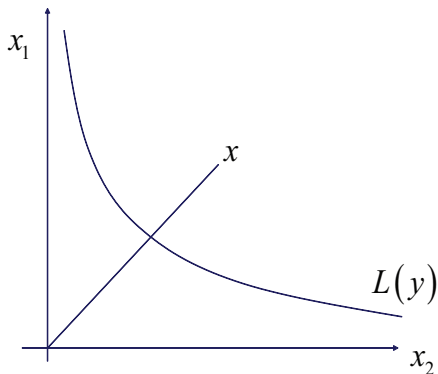
Distance Functions and Risk Aversion

- One tool used in production analysis is the distance function. In production theory, the distance function is typically defined as the fraction of input that can be taken away in a way that still satisfies a level set (or level of production).
- Specifically,

$$D_I(x, y) = \max_{\theta} \left\{ \theta : \left(\frac{x}{\theta} \right) \in L(y) \right\} \quad (8)$$

where θ is a scalar distance to be determined by the formulation, x is a vector of inputs, and $L(y)$ is a level set (a set that produces at least output y).

Figure : Distance Function in Input Space

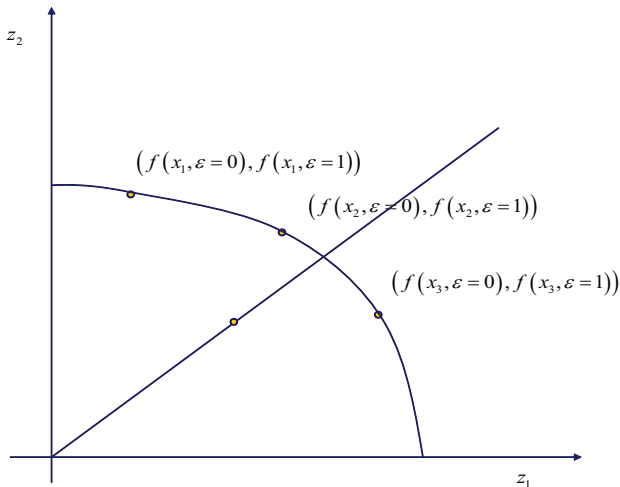


- Using a similar justification, Chambers and Quiggin propose a *state-contingent output-distance function*

$$O(z, x) = \inf \left\{ \theta > 0 : \frac{z}{\theta} \in Z(x) \right\} \quad (9)$$

Using this definition, for the point is on the interior of the *state-contingent output function*.

Figure : State-Contingent Output Function



- This transformation has something to say about choice of outputs across states. Specifically, the rate at which output in one state can be traded for output in another state can be formulated as

$$\frac{\frac{\partial z_2}{\partial z_1}}{\frac{\partial O(z, x)}{\partial z_2}} = \frac{\frac{\partial O(z, x)}{\partial z_1}}{\frac{\partial O(z, x)}{\partial z_2}} \quad (10)$$