

Real Options: Lecture XXXIX

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1 Real Option Valuation

Real Option Valuation

- Moss, Pagano, and Boggess. "Ex Ante Modeling of the Effect of Irreversibility and Uncertainty on Citrus Investments"
 - Traditional courses in financial management state that an investment should be undertaken if the Net Present Value of the investment is positive.
 - However, firms routinely fail to make investments that appear profitable considering the time value of money.
 - Several alternative explanation for this phenomenon have been proposed. However, the most fruitful involves risk.
 - Integrating risk into the decision model may take several forms from the Capital Asset Pricing Model to stochastic net present value.
 - However, one avenue which has gained increased attention during the past decade is the notion of an investment as an option.

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 - Several characteristics of investments make the use of option pricing models attractive.
 - In most investments, investors can be construed to have limited liability with the distribution being truncated at the loss the entire investment.
 - Alternatively, Dixit and Pindyck have pointed out that the investment decision is very seldomly a now or never decision. The decision maker may simply postpone exercising the option to invest.

- Derivation of the value of waiting.
 - As a first step in the derivation of the value of waiting, we consider an asset whose value changes over time according to a geometric Brownian motion stochastic process

$$dV = \alpha Vdt + \sigma Vdz \quad (1)$$

- Given the stochastic process depicting the evolution of asset values over time, we assume that there exists a perfectly correlated asset that obeys a similar process

$$\begin{aligned} dx &= \mu xdt + \sigma xdz \\ \mu &= r + \phi \rho_{vm} \sigma \end{aligned} \quad (2)$$

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- Comparing the two stochastic processes leads to a comparison of α and μ .
 - The relationship between these two values gives rise to the execution of the option.
 - Defining $\delta = \mu - \alpha$ to the dividend associated with owning the asset.
 - α is the capital gain while μ is the "operating return".
- If δ is less than or equal to zero, the option will never be exercised.
- Thus, $\delta > 0$ implies that the operating return is greater than the capital gain on a similar asset.

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- Next, we construct a riskless portfolio containing one unit of the option to some level of short sale of the original asset

$$P = F(V) - F_V(V) V \quad (3)$$

P is the value of the riskless portfolio, $F(V)$ is the value of the option, and $F_V(V)$ is the derivative of the option price with respect to value of the original asset.

- Dropping the V s and differentiating the riskfree portfolio we obtain the rate of return on the portfolio. To this differentiation, we append two assumption
 - The rate of return on the short sale over time must be $-\delta V$ (the short sale must pay at least the expected dividend on holding the asset).
 - The rate of return on the riskfree portfolio must be equal to the riskfree return on capital $r(F - F_V V)$.

$$dF - F_V dV - \delta V F_V dt = r(F - F_V V) dt \quad (4)$$

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 - Combining this expression with the original geometric process and applying Ito? Lemma we derive the combined zero-profit and zero-risk condition

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + (r - \delta) V F_V - rF = 0 \quad (5)$$

In addition to this differential equation we have three boundary conditions

$$F(0) = 0, F(V^*) = V^* - I, F_V(V^*) = 1 \quad (6)$$

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 - The solution of the differential equation with the stated boundary conditions is

$$\begin{aligned} F(V) &= \alpha V^\beta \\ \alpha &= \frac{V^* - I}{V^{*\beta}} \\ V^* &= \frac{\beta}{\beta - 1} I \end{aligned} \tag{7}$$

$$\beta = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \left\{ \left[\frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]^2 + 2 \frac{r}{\sigma^2} \right\}^{\frac{1}{2}}$$

- β then simplifies to

$$\beta = \frac{1}{2} \left\{ 1 + \left(1 + \frac{8r}{\sigma^2} \right)^{\frac{1}{2}} \right\} \tag{8}$$

- Estimating β
 - In order to incorporate risk into an investment decision using the Dixit and Pindyck approach we must estimate σ .
 - This one approach to estimating s is through simulation. Specifically, simulating the stochastic Net Present Value of an investment as

$$V_t = \sum_{i=1}^{t+N} \frac{CF_{t_i}}{(1+r)^i} \quad (9)$$

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 - Converting this value to an infinite streamed investment then involves deriving the Annualized Net Present Value for the finite stream

$$APV_t = \frac{V_t}{1 - \frac{1}{(1+r)^N}} \cdot \frac{1}{r} \quad (10)$$

Then we define the infinite present value

$$V_t^* = \frac{APV_t}{r} \quad (11)$$

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 - The parameters of the stochastic process can then be estimated by

$$\begin{aligned}\frac{dV}{V} &= d \ln(V) \\ &= \ln(V_{t+1}) - \ln(V_t)\end{aligned}\tag{12}$$

