

Increasing Risk: Lecture XXVIII

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1 Literature Required

Literature Required

- Over the next several lectures we will be developing the notion of stochastic dominance with respect to a function.
- Meyer is the primary contributor to the basic literature. The primary readings will be
 - Meyer, J. 1975. Increasing Risk. *Journal of Economic Theory* 11: 119–132.
 - Meyer, J. 1977. Choice Among Distributions. *Journal of Economic Theory* 14: 326–336.
 - Meyer, J. 1977. Second Degree Stochastic Dominance with Respect to a Function. *International Economic Review* 18: 477–487.

- However, in working through Meyer's articles, we need a couple of concepts from:
 - Diamond, P.A. and J.E. Stiglitz. 1974. Increases in Risk and in Risk Aversion. *Journal of Economic Theory* 8: 337–360.
 - Pratt, J. 1964. Risk Aversion in the Small and Large. *Econometrica* 23: 122-136.
 - Rothschild, M. and J.E. Stiglitz. 1970. Increasing Risk I, A Definition. *Journal of Economic Theory* 2: 225–243.

- Assuming x and y are random variables with cumulative distributions F and G , respectively.
- In general we will label the cumulative distribution functions in such a way that G will be at least as risky as F .
- Further, the choice among cumulative distributions will be made on the basis of expected utility.

$$\int_0^1 u(x) dF(x) \geq \int_0^1 u(x) dG(x) \quad (1)$$

- Rothschild and Stiglitz showed three ways of defining increasing risk:
 - $G(x)$ is at least as risky as $F(x)$ if $F(x)$ is preferred to or indifferent to $G(x)$ by all risk averse agents.
 - $G(x)$ is at least as risky as $F(x)$ if $G(x)$ can be obtained from $F(x)$ by a sequence of steps which shift weight from the center of $F(x)$ to its tails without changing the means.
 - $G(y)$ is at least as risky as $F(x)$ if y is a random variable that is equal in the distribution of x plus some random noise.

- Rothschild and Stiglitz found that necessary and sufficient conditions on the cumulative distributions $F(x)$ and $G(x)$ for $G(x)$ to be at least as risky as $F(x)$ are:

$$\int_0^1 [G(x) - F(x)] dx \geq 0 \forall y \in [0, 1] \quad (2)$$
$$\int_0^1 [G(x) - F(x)] dx \neq 0 \text{ for at least one } x$$

- Rothschild and Stiglitz showed that this definition yields a partial ordering over the set of cumulative distributions in terms of riskiness.
- The ordering is partial in two senses:
 - Only cumulative distributions of the same mean can be ordered.
 - Not all distributions with the same mean can be ordered.

- Thus, a necessary but not sufficient condition for one distribution to be riskier than another by Rothschild and Stiglitz is that there means be equal.

- Diamond and Stiglitz extended Rothschild and Stiglitz defining increasing risk as: $G(x)$ is at least as risky as $F(x)$ if $G(x)$ can be obtained from $F(x)$ by a sequence of steps, each of which shifting weight from the center of $F(x)$ to its tails while keeping the expectation of the utility function, $u(x)$ is constant.
- Definition Based on Unanimous Preference:
 - Rothschild and Stiglitz 's first definition defines $G(x)$ to be as risky as $F(x)$ if $F(x)$ is preferred or indifferent to $G(x)$ by all risk averse agents.
 - In other words, $F(x)$ is unanimously preferred by the class of agents known as risk averse.

- Restating the general principle we could say that $G(x)$ is at least as risky as $F(x)$ if $F(x)$ is unanimously preferred or indifferent to $G(x)$ by all agents who are at least as risk averse as a risk neutral agent.
- Definition 1: Cumulative distribution $G(x)$ is at least as risky as cumulative distribution $F(x)$ if there exists some agent with a strictly increasing utility function $u(x)$ such that for all agents more risk averse than he, $F(x)$ is preferred or indifferent to $G(x)$.

- A Mean Preserving Spread

- A mean preserving spread is a function which when added to the density function transfers weight from the center of the density function to its tails without changing the mean.
- Formally, $s(x)$ is a mean preserving spread if

$$\int_0^1 s(x) dx = 0 \quad (3)$$

- $s(x)$ changes sign at most twice in $(0, 1)$.
- $s(x) \geq 0 \forall x \leq z$ for some $z \in (0, 1)$.
- $s(x_0) > 0$, for some $x_0 \in [0, z)$.

- Definition 2: Cumulative distribution $G(x)$ is at least as risky as cumulative distribution $F(x)$ if $G(x)$ can be obtained from $F(x)$ by a finite sequence of cumulative distributions

$$\begin{aligned} F_1(x) &= s_1 F(x) \\ F_2(x) &= s_2 F_1(x) \\ &\vdots \\ G(x) &= s_n(x) F_{n-1}(x) \end{aligned} \quad (4)$$

where each $F_i(x)$ differs from $F_{i-1}(x)$ by a single spread.