

Development of FSD and SSD: Lecture XXVII

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1 Efficiency Criteria

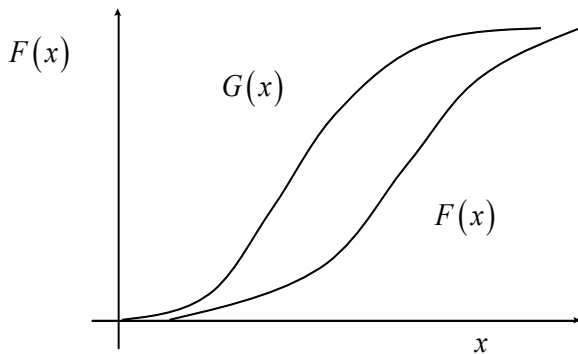
The Concept of an Efficiency Criteria

- An efficiency criteria is a decision rule for dividing alternatives into two mutually exclusive groups: efficient and inefficient.
 - If an alternative is in the efficient group, then it is one that an investor may choose.
 - An inefficient investment will not be chosen by any investor regardless of individual risk preferences.

- From an economic standpoint, the criteria should be related to general notions of utility or preferences.
 - In general, the more global the preference, the less discerning the criteria (i.e. the fewer alternatives eliminated).
 - A smaller efficient set requires more stringent requirements on preferences.

- The most general efficiency criteria relies only on the assumption that utility is nondecreasing in income, or the decision maker prefers more of at least one good to less.
 - **FSD Rule:** Given two cumulative distribution functions F and G , an option F will be preferred to the second option G by FSD independent of concavity if $F(x) \leq G(x)$ for all return x with at least one strict inequality.
 - Intuitively, this rule states that one alternative F will dominate G if its cumulative distribution function always lies to the left of G 's as depicted in Figure 1.

Figure: First Degree Stochastic Dominance



- Continued

- Mathematically, FSD is dependent on the integrals of the utility function times each alternative distribution function

$$E_F U(x) = \int_a^b U(x) f(x) dx \text{ versus } E_G U(x) = \int_a^b U(x) g(x) dx \quad (1)$$

- Note that the utility function is the same for each investment alternative, but the distribution function changes. If investment F dominates investment G , then the difference, Δ , defined as

$$\Delta = E_F U(x) - E_G U(x) = \int_a^b [f(x) - g(x)] U(x) dx \quad (2)$$

- Continued
 - Integrating by parts

$$\Delta = (U(x) [F(x) - G(x)]|_a^b + \int_a^b [G(x) - F(x)] U'(x) dx \quad (3)$$

- Second Degree Stochastic Dominance
 - Building on FSD, second degree stochastic dominance SSD invokes risk aversion by inferring that the utility function is concave, implying that the second derivative of the utility function is negative.
 - **SSD Rule** A necessary and sufficient condition for an alternative F to be preferred to a second alternative G by all risk averse decision makers is that

$$\begin{aligned} \int_{-\infty}^x F(z) dz &\leq \int_{-\infty}^x G(z) dz \\ \int_{-\infty}^x [G(z) - F(z)] dz &\geq 0 \end{aligned} \tag{4}$$

- Continued
 - Again, integrating by parts

$$\begin{aligned} \left(U'(x) \int_a^b [G(x) - F(x)] dx \right) \Big|_a^b + \int_a^b \left(-U''(x) \int_a^x [G(z) - F(z)] dz \right) dx \\ \Rightarrow \int_a^x [G(z) - F(z)] dz \leq 0 \end{aligned} \quad (5)$$

- Continued
 - Graphically, another explanation of SSD can be determined by: Alternative F dominates alternative G for all risk averse individuals if the cumulative area under F exceeds the area under the cumulative distribution function G for all values x , or if the cumulative area between F and G is non-negative for all x .

Table: Crop Yields

	Corn	Soybean	Wheat
1	133.9547	117.0221	106.7415
2	160.9626	154.2518	118.1192
3	136.3654	135.6167	102.3978
4	145.1248	133.0063	127.3458
5	134.4932	122.7777	115.3229
6	122.7883	114.4257	108.2281
7	123.0762	121.1467	111.1333
8	100.5129	102.8930	118.9812
9	129.9954	143.0956	114.0733
10	134.2391	134.0585	116.8623

Table: Comparison of Cumulative Density Functions

Income	CDF(C)	CDF(S)	CDF(W)	C-S	C-W	S-W
100.51	0.1	0.0	0.0	0.1	0.1	0.0
102.40	0.1	0.0	0.1	0.1	0.0	-0.1
102.89	0.1	0.1	0.1	0.0	0.0	0.0
106.74	0.1	0.1	0.2	0.0	-0.1	-0.1
108.23	0.1	0.1	0.3	0.0	-0.2	-0.2
111.13	0.1	0.1	0.4	0.0	-0.3	-0.3
114.07	0.1	0.1	0.5	0.0	-0.4	-0.4
114.43	0.1	0.2	0.5	-0.1	-0.4	-0.3
115.32	0.1	0.2	0.6	-0.1	-0.5	-0.4
116.86	0.1	0.2	0.7	-0.1	-0.6	-0.5
117.02	0.1	0.3	0.7	-0.2	-0.6	-0.4
118.12	0.1	0.3	0.8	-0.2	-0.7	-0.5
118.98	0.1	0.3	0.9	-0.2	-0.8	-0.6

Figure: Empirical First Degree Stochastic Dominance

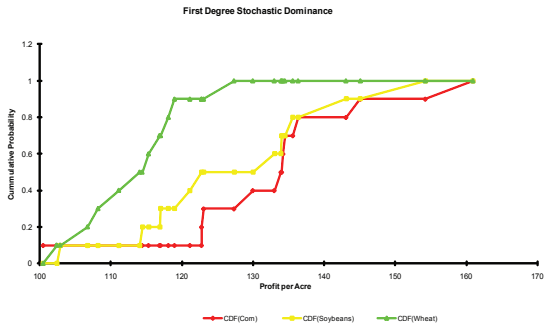


Table: Area Under the Cumulative Density Functions

Gross Margin	Corn	Soybean	Wheat	C-S	C-W	S-W
100.51	0.1885	0.0000	0.0000	0.1885	0.1885	0.0000
102.40	0.2380	0.0000	0.0495	0.2380	0.1885	-0.0495
102.89	0.6229	0.3849	0.4344	0.2380	0.1885	-0.0495
106.74	0.7715	0.5335	0.7317	0.2380	0.0398	-0.1982
108.23	1.0620	0.8240	1.6032	0.2380	-0.5412	-0.7792
111.13	1.3560	1.1180	2.7793	0.2380	-1.4232	-1.6612
114.07	1.3913	1.1533	2.9555	0.2380	-1.5642	-1.8022
114.43	1.4810	1.3327	3.4041	0.1483	-1.9231	-2.0714
115.32	1.6349	1.6406	4.3277	-0.0056	-2.6928	-2.6871
116.86	1.6509	1.6726	4.4396	-0.0216	-2.7886	-2.7670
117.02	1.7606	2.0017	5.2075	-0.2410	-3.4469	-3.2059
118.12	1.8468	2.2603	5.8971	-0.4134	-4.0503	-3.6368
118.98	2.0634	2.9099	7.8461	-0.8465	-5.7827	-4.9362
121.15	2.2265	3.5623	9.3140	-1.3358	-7.0875	-5.7517
122.78	2.2275	3.5676	9.3235	-1.3401	-7.0959	-5.7559
122.79	2.2851	3.7116	9.5826	-1.4264	-7.2975	-5.8710

Table: Area Under the Cumulative Density Functions

Gross Margin	Corn	Soybean	Wheat	C-S	C-W	S-W
123.08	3.5660	5.8464	13.4253	-2.2804	-9.8593	-7.5789
127.35	4.3609	7.1711	16.0748	-2.8103	-11.7139	-8.9037
130.00	5.5653	8.6766	19.0858	-3.1114	-13.5205	-10.4092
133.01	5.9446	9.2457	20.0342	-3.3010	-14.0896	-10.7885
133.95	5.9965	9.3079	20.1379	-3.3114	-14.1414	-10.8300
134.06	6.0868	9.4343	20.3185	-3.3475	-14.2317	-10.8842
134.24	6.2393	9.6122	20.5727	-3.3730	-14.3334	-10.9604
134.49	7.0257	10.3987	21.6962	-3.3730	-14.6704	-11.2975
135.62	7.5498	10.9976	22.4449	-3.4478	-14.8950	-11.4472
136.37	12.9339	16.3818	29.1750	-3.4478	-16.2411	-12.7933
143.10	14.5573	18.2081	31.2043	-3.6508	-16.6469	-12.9962
145.12	22.7716	26.4224	40.3313	-3.6508	-17.5596	-13.9089
154.25	28.8114	33.1332	47.0421	-4.3218	-18.2307	-13.9089

Figure: Second Degree Stochastic Dominance

