

Overview of Stochastic Dominance: Lecture XXVI

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- 1 Expected Utility Versus Risk Efficiency
- 2 Stochastic Dominance

Expected Utility Versus Risk Efficiency

- In this course, we started with the precept that individual's choose between actions or alternatives in a way that maximizes their expected utility.
 - **Ordering and transitivity:** A person either prefers one of two risky prospects a_1 and a_2 or is indifferent between them. Further if the individual prefers a_1 to a_2 and a_2 to a_3 , then he prefers a_1 to a_3 .
 - **Continuity:** If a person prefers a_1 to a_2 to a_3 , then there exists some subjective probability level $Pr[a_1]$ such that he is indifferent between the gamble paying a_1 with probability $Pr[a_1]$ and a_3 with probability $1 - Pr[a_1]$ which leaves him indifferent with a_2 .
 - **Independence:** If a_1 is preferred to a_2 , and a_3 is any other risky prospect, a lottery with a_1 and a_3 outcomes will be preferred to a lottery with a_2 and a_3 outcomes when $Pr[a_1] = Pr[a_2]$. In other words, preference between a_1 and a_2 is independent of a_3 .

- However, some literature has raised questions regarding the adequacy of these assumptions:
 - Allais (1953) raised questions about the axiom of independence.
 - May (1954) and Tversky (1969) questioned the transitivity of preferences.

- These studies question whether preferences under uncertainty are adequately described by the traditional expected utility framework. One alternative is to develop risk efficiency criteria rather than expected utility axioms.
 - Risk efficiency criteria are an attempt to reduce the collection of all possible alternatives to a smaller collection of risky alternatives that contain the optimum choice.
 - One example was the mean-variance derivation of optimum portfolios.
 - The EV frontier contained the set of possible portfolios such that no other portfolio could be constructed with a higher return with the same risk measured as the variance of the portfolio.
 - It was our contention that this efficient set contained the utility maximizing portfolio. In addition, we derived the conditions which demonstrated how the EV framework was consistent with expected utility.

- Instead of expected utility justifying risk efficiency, we are now interested in the derivation of risk efficiency measures under their own right.
- An alternative justification of risk efficiency measures involves the scenario where the individual's risk preferences are difficult to elicit.

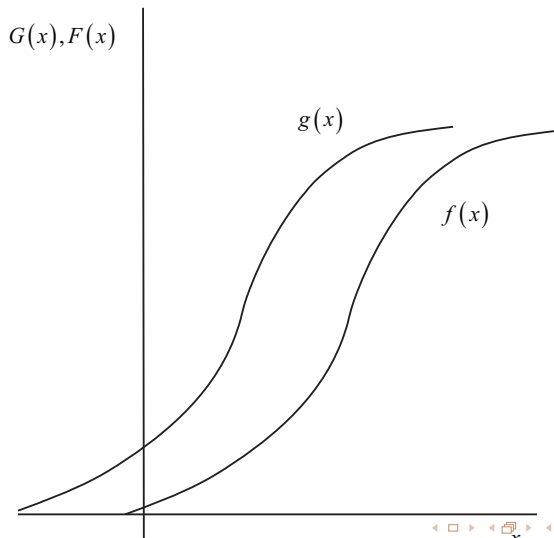
Stochastic Dominance

- One of the most frequently used risk efficiency approaches is stochastic dominance. To demonstrate the concept of stochastic dominance, let's examine the simplest form of stochastic dominance (first order stochastic dominance).
 - To develop first order stochastic dominance, let us assume that the decision maker is faced with two alternative investments, a and b .
 - Assume that the probability density function for alternative a can be characterized by the probability density function $f(x)$.
 - Similarly, assume that the return on investment b is associated with the probability density function $g(x)$.
 - Investment a is said to be first order dominant of investment b if and only if

$$\int_{-\infty}^x g(s) ds \geq \int_{-\infty}^x f(s) ds \quad (1)$$

for all x as depicted in Figure 1.

Figure: Stochastic Dominance



- Continued
 - Thus, investment a is always more likely to yield a higher return. Intuitively, one investment is going to dominate the other investment if their cumulative distribution functions do not cross.
- This very basic criteria would appear noncontroversial, however, it is not very discerning. Taking the test data set presented in Table 1

Table: Probability of Payoffs for Three Alternatives

Probability	Alt 1	Alt 2	Alt 3
0.1	100.5129	102.8930	102.3978
0.2	122.7883	114.4257	106.7415
0.3	123.0762	117.0221	108.2281
0.4	129.9954	121.1467	111.1333
0.5	133.9547	122.7777	114.0733
0.6	134.2391	133.0063	115.3229
0.7	134.4932	134.0585	116.8623
0.8	136.3654	135.6167	118.1192
0.9	145.1248	143.0956	118.9812
1.0	160.9626	154.2518	127.3458

Table: Probability Dependent Table

Payoff	Alt 1	Alt 2	Alt 3
100.51286	0.1	0.0	0.0
102.39779	0.1	0.0	0.1
102.89301	0.1	0.1	0.1
106.74153	0.1	0.1	0.2
108.22805	0.1	0.1	0.3
111.13325	0.1	0.1	0.4
114.07334	0.1	0.1	0.5
114.42573	0.1	0.2	0.5
115.32292	0.1	0.2	0.6
116.86233	0.1	0.2	0.7
117.02213	0.1	0.3	0.7
118.11924	0.1	0.3	0.8
118.98116	0.1	0.3	0.9
121.14674	0.1	0.4	0.9
122.77773	0.1	0.5	0.9

Table: Probability Dependent Table

Payoff	Alt 1	Alt 2	Alt 3
122.78827	0.2	0.5	0.9
123.07618	0.3	0.5	0.9
127.34582	0.3	0.5	1.0
129.99535	0.4	0.5	1.0
133.00632	0.4	0.6	1.0
133.95473	0.5	0.6	1.0
134.05845	0.5	0.7	1.0
134.23908	0.6	0.7	1.0
134.49320	0.7	0.7	1.0
135.61670	0.7	0.8	1.0
136.36541	0.8	0.8	1.0
143.09556	0.8	0.9	1.0
145.12480	0.9	0.9	1.0
154.25181	0.9	1.0	1.0
160.96262	1.0	1.0	1.0

Figure: Empirical Cumulative Distribution Function

