

The Farm Portfolio Problem – Target MOTAD and Discrete Sequential Stochastic Programming: Lecture XXV

Charles B. Moss

September 10, 2026

- 1 Target MOTAD
- 2 Discrete Sequential Stochastic Programming
- 3 Chance Constrained Programming

Target MOTAD

- The target MOTAD model is a two-attribute risk and return model.
- Return is measured as the sum of the expected return of each activity multiplied by the activity level.
- Risk is measured as the expected sum of the negative deviations of the solution results from a target-return level.
- Risk is then varied parametrically so that a risk-return frontier can be traced out.

- Mathematically, the model is stated as

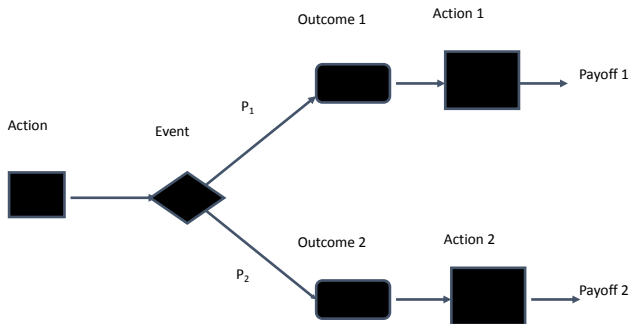
$$\begin{aligned} \max_x E(z) &= \sum_{j=1}^n c_j x_j \\ \text{s.t. } \sum_{j=1}^n a_{ij} x_j &\leq b_i \\ T - \sum_{j=1}^n c_{rj} x_j - y_r &\leq 0 \\ \sum_{j=1}^n p_r y_r &= \lambda \end{aligned} \tag{1}$$

Discrete Sequential Stochastic Programming

- Target MOTAD, direct expected utility, and even MOTAD begin to develop the concept of constraints being stochastic or met with some level of probability.
 - In target MOTAD, income under a certain state exceeds the target level of income with some probability.
 - In direct expected utility maximization the level of wealth transferred to the objective function was represented by a constraint which had some level of probability.
 - In MOTAD, we minimized the expected negative deviations which implied stochastic constraints.
 - However, in each of these cases, the primary impact of stochastic constraints was on the objective function or some threshold level of risk (as was the case in target MOTAD).

- The variant of model that we want to develop is referred to as Discrete Sequential Stochastic Programming (DSSP), although other names have been attributed to it.
- This work grows out of work by Cocks, and focuses on decision processes which are strung out over a discrete number of decision periods.

Discrete Sequential Decision Tree



- At a discrete point in the future, the farmer has to make a decision, for example a stocking rate on cattle.
- Given this first round decision and a random outcome, such as rainfall, there is then a subsequent decision to be made, for example whether to sell cattle or buy feed.
- Each state occurs with a given level of probability and each "node" can contribute to the objective function.

$$\begin{aligned}
 \max_x \quad & -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 + c_5x_5 - c_6x_6 + c_7x_7 - c_8x_8 - c_9x_9 \\
 & -s_{11}x_1 + u_{11}x_2 - f_{11}x_3 \\
 & -s_{21}x_1 + u_{21}x_2 - f_{21}x_4 \\
 & \quad -x_2 \quad \quad \quad + x_5 - x_6 + x_7 \\
 & -s_{12}x_1 \quad \quad \quad + u_{12}x_7 - f_{12}x_8 \\
 & -s_{22}x_1 \quad \quad \quad + u_{22}x_7 - f_{22}x_9
 \end{aligned} \tag{2}$$

- In this model x_1 represents the acres of wheat planted, x_2 is the number of stockers purchased, x_3 the tons purchased under outcome 1, and x_4 the tons of feed under outcome 2.
- The first two equations, then, simply balance the feed requirements under each state of nature.
 - For example, if there is good rainfall in state 1, then more grazing will be produced by the wheat, x_2 , and less feed will have to be purchased than in state 2.
 - c_1 and c_2 are then the cost of feed in each state weighted by the probability of that state.

- The third equation then transfers the cattle purchased into the next decision period.
 - x_5 is a variable modeling the number of stockers sold, while x_6 models any additional stockers purchased.
 - The total number of stockers in the next production period is x_7 .
 - Given the number of cattle transferred into the next period the feed balance relationships determine the level of feed that must be purchased.

Chance Constrained Programming

- The DSSP problem above assumes that the possible outcomes can be represented in a finite number of states, although several pieces of applied research have examined the efficiency of approximating the moments of a continuous distribution with a finite number of points.
- An alternative would be to constrain the probability.
 - For example, assume that you want to constrain the probability that profit will be less than a fixed level T (to borrow the target MOTAD concept).
 - Mathematically, this constraint becomes:

$$P[X < x^*] \leq T \quad (3)$$

- Under normality, we can transform this constraint via the confidence interval:

$$\begin{aligned}
 x' \mu - \alpha_{0.05} \sqrt{x' \Omega x} &\geq T \\
 \alpha_{0.05} &\geq \frac{x' \mu}{\sqrt{x' \Omega x}} - \frac{T}{\sqrt{x' \Omega x}} \\
 \sum_j x_{ij} \mu_{ij} D^* \left[\left(\sum_k \sigma_{ik}^2 x_j^2 \right)^{1/2} \right] &\leq b_i
 \end{aligned} \tag{4}$$