

The Farm Portfolio Problem – MOTAD and Direct Expected Utility: Lecture XXIV

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1 MOTAD

2 Focus-Loss

3 Direct Expected Utility

MOTAD

- Hazell, P.B.R. 1971. A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty. *American Journal of Agricultural Economics* 53(1): 53-62.
- Hazell's approach is two fold. He first sets out to develop review expected value/variance as a good methodology under certain assumptions.
- Then he raises two difficulties.
 - The first difficulty is the availability of code to solve the quadratic programming problem implied by EV.
 - The second problem is the estimation problem. Specifically, the data required for EV are the mean and the variance matrix.

Approximation to Variance

- The variance of a particular farming plan can be expressed as

$$\sum_{j=1}^n \sum_{i=1}^n x_i x_j \left[\frac{1}{T-1} \sum_{t=1}^T (c_{tj} - g_j) (c_{tk} - g_k) \right] \quad (1)$$

$$\sigma^2 = \frac{1}{T-1} \sum_{t=1}^T \left[\sum_{j=1}^N c_{tj} x_j - \sum_{j=1}^n g_j x_j \right]^2$$

- Hazell suggests replacing this objective function with the mean absolute deviation

$$A = \frac{1}{T} \sum_{t=1}^T \left| \sum_{j=1}^n (c_{tj} - g_j) x_j \right| \quad (2)$$

- Instead of minimizing the variance of the farm plan subject to an income constraint, you can minimize the absolute deviation subject to an income constraint.
- Another formulation for this objective function is to let each observation t be represented by a single row

$$\begin{aligned} y_t &= \sum_{j=1}^n (c_{tj} - g_j) x_j \\ y_t^+ + y_t^- &= \sum_{j=1}^n (c_{tj} - g_j) x_j \end{aligned} \tag{3}$$

- Two sided errors

$$\begin{aligned}
 \min_x \quad & TA = \sum_{t=1}^T (y_t^+ + y_t^-) \\
 \text{s.t.} \quad & \sum_{j=1}^n (c_{tj} - g_j) x_j - y_t^+ + y_t^- = 0 \\
 & \sum_{j=1}^n g_j x_j = \lambda \sum_{j=1}^n a_{ij} x_j \leq b_i
 \end{aligned} \tag{4}$$

- One sided errors

$$\begin{aligned}
 \min_x TA &= \sum_{t=1}^T y_t^- \\
 \text{s.t. } \sum_{j=1}^n (c_{tj} - g_j) x_j + y_t^- &= 0 \\
 \sum_{j=1}^n g_j x_j &= \lambda \sum_{j=1}^n a_{ij} x_j \leq b_i
 \end{aligned} \tag{5}$$

Hazell's Florida Farm

Obs.	Carrots	Celery	Cucumbers	Peppers
1	292	-128	420	579
2	179	560	187	639
3	114	648	366	379
4	247	544	249	924
5	426	182	322	5
6	259	850	159	569

Deviations Florida Farm

Obs.	Carrots	Celery	Cucumbers	Peppers
1	39	-571	136	63
2	-74	117	-97	123
3	-139	205	82	-137
4	-6	101	-35	408
5	173	-261	38	-511
6	6	407	-125	53

$$\begin{aligned}
 \min_x \quad & y_1^- + y_2^- + y_3^- + y_4^- + y_5^- + y_6^- \\
 & x_1 + x_2 + x_3 + x_4 \leq 200 \\
 & 25x_1 + 36x_2 + 27x_3 + 87x_4 \leq 10,000 \\
 & -x_1 + x_2 - x_3 + x_4 \leq 0 \\
 & 39x_1 - 571x_2 + 136x_3 + 63x_4 + y_1^- \geq 0 \\
 & -74x_1 + 117x_2 - 97x_3 + 123x_4 + y_2^- \geq 0 \\
 & -139x_1 + 205x_2 + 82x_3 - 137x_4 + y_3^- \geq 0 \\
 & -5x_1 + 101x_2 - 35x_3 + 408x_4 + y_4^- \geq 0 \\
 & 173x_1 - 261x_2 + 38x_3 - 511x_4 + y_5^- \geq 0 \\
 & 6x_1 + 407x_2 - 125x_3 + 53x_4 + y_6^- \geq 0 \\
 & 253x_1 + 443x_2 + 284x_3 + 516x_4 = \lambda
 \end{aligned} \tag{6}$$

Focus-Loss

- Two factors make Focus-Loss acceptable
 - First, like Hazell's MOTAD, the Focus-Loss problem is solvable using linear programming.
 - Second, Focus-Loss has a direct appeal in that it focuses attention on survivability
- The first step in the Focus-Loss methodology is to define the maximum allowable loss

$$L = E[z] - z_c = \sum_{j=1}^n E(c_j) x_j - E(F) - z_c \quad (7)$$

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|---|--|
| • L - Maximum allowable loss | • $E[c_j]$ - Expected income from crop j |
| • $E[z]$ - Expected income for the farm | • x_j - Level of the j^{th} crop (activity) |
| • z_c - Required cash income | • $E[F]$ - Expected level of fixed cost |

- Given this definition, the next step is to define the maximum deficiencies or loss arising from activity j

$$r_j = E[c_j] - r_j^* \quad (8)$$

where r_j^* is the worst expected outcome.

- For example, a crop failure may give an r_j of -\$100 which would represent your planting cost.
- Given this potential loss, the Focus-Loss scenario is based on restricting the largest expected loss to be above some stated level

$$r_j x_j \leq \frac{L}{K} \quad (9)$$

$$\begin{aligned}
 \max_x \quad & 72x_1 + 53.4x_2 + 88.8x_3 - 200 \\
 & x_1 + x_2 + x_3 \leq 12 \\
 & x_1 + \quad + x_3 \leq 8 \\
 & 30x_1 + 20x_2 + 40x_3 \leq 400 \\
 & 5x_1 + 5x_2 + 8x_3 \leq 80 \\
 & 60x_1 - \frac{1}{3}L \leq 0 \\
 & \quad 44.5x_2 - \frac{1}{3}L \leq 0 \\
 & \quad \quad 74x_3 - \frac{1}{3}L \leq 0
 \end{aligned} \tag{10}$$

- The choice of $k = 3$ is somewhat arbitrary.
- Two points about Focus-Loss
 - Allowing $L \rightarrow -\infty$ the Focus-Loss solution is the profit maximizing solution.
 - L can become large enough to make the linear programming problem infeasible.
- A better justification for k
 - One alternative for setting k results from the notion that

$$r_j^* = \mu_j - t_p \sigma_j. \quad (11)$$

- Thus, if we let t_p be -1.96, the maximum loss would be $1.96 \sigma_j$

$$(\sigma_j t_p) x_j \leq \frac{L}{k} \quad (12)$$

Direct Expected Utility

- We have been discussing several alternatives to utility maximization based on efficiency criteria or *ad hoc* specifications of risk aversion as in the case of focus-loss.
- One alternative is direct use of expected utility.

Data for Direct Utility Maximization

Obs	Corn	Soybean	Wheat
1	176.24	94.81	97.09
2	232.93	114.39	120.18
3	273.01	144.5	108.75
4	221.59	114.32	87.48
5	-7.87	97.22	100.46
6	247.59	126.41	108.34
7	226.79	113.49	98.16
8	250.11	123.27	107.6
9	255.99	136.15	102.81
10	246.91	131.04	104.68
Average	212.33	119.56	103.56

- Total acres do not exceed 1280.
 - Annual profit of \$271,782.
 - Amortizing this amount into perpetuity using a discount rate of 15% yields a total value of \$ 1,811,880.
- Assuming the debt-to-asset position of the farm is 60%, the value of the asset represents equity of \$724,752 and debt of \$1,087,130.
- Assuming an interest rate of 12.5% yields an annual cash flow requirement of \$135,891 to cover the interest payments.
- Assuming a family living requirement of \$50,000 yields a minimum cash requirement of \$185,891.

$$W_i = 176.24x_1 + 94.81x_2 + 97.09x_3 + 538,861. \quad (13)$$

$$\begin{array}{rcl}
 \max_x & \frac{1}{10} \frac{W_1^b}{b} + \frac{1}{10} \frac{W_2^b}{b} + \dots + \frac{1}{10} \frac{W_{10}^b}{b} & \\
 & x_1 + x_2 + x_3 & \leq 1,280 \\
 & -176.2x_1 - 93.8x_2 - 97.1x_3 + W_1 & = 538,861 \\
 & -239.9x_1 - 114.4x_2 - 120.2x_3 + W_2 & = 538,861 \\
 & \vdots & \\
 & -246.9x_1 - 131.0x_2 - 104.7x_3 & W_{10} = 538,861 \\
 & & (14)
 \end{array}$$

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