

Meyer's Location Scale: Lecture XV

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1 Definition of Location-Scale

- Class I
- Expected Utility

2 Properties of Expected Utility

Definition of Location-Scale

- *Definition:* Two cumulative distributions functions $G_1(.)$ and $G_2(.)$ are said to differ only by location and scale parameters and if $G_1(x) = G_2(\alpha + \beta x)$ with $\beta > 0$.
- Several two-parameter families satisfy this property such as the normal and the uniform distributions.
- Assume that there exists various choice sets, Y_i , that only differ by location and scale parameters.

$$X_i = \frac{Y_i - \mu_i}{\sigma_i} \quad (1)$$

Linkage to Expected Utility

- The expected utility of this alternative can then be derived as

$$\begin{aligned} EU[Y_i] &= \int_a^b u(\mu_i + \sigma_i x) dF(x) \\ &\equiv V(\sigma_i, \mu_i) \end{aligned} \quad (2)$$

- The first step in examining the properties of $V(\sigma, \mu)$ is to examine its partial derivatives

$$\begin{aligned} V_\sigma(\sigma, \mu) &= \int_a^b u'(\mu + \sigma x) x dF(x) \\ &= \int_a^b \left(u''(\mu + \sigma x) \int_a^x x dF(x) \right) dx \\ V_\mu(\sigma, \mu) &= \int_a^b u'(\mu + \sigma x) dF(x) \end{aligned} \quad (3)$$

- *Property 1:* $V_{\mu}(\sigma, \mu) > 0$ for all μ and all $\sigma > 0$ if and only if $u'(\alpha + \beta x) \geq 0$ for all $\alpha + \beta x$.
- Property 1 states that increases in increase $V(\cdot)$ if the marginal utility $u'(\cdot)$ is positive. Similarly, increases in reduce $V(\cdot)$ if the utility function is concave. Defining $S(\cdot, \cdot)$ as

$$S(\sigma, \mu) = \frac{-V_{\sigma}(\sigma, \mu)}{V_{\mu}(\sigma, \mu)} = \frac{\int_a^b \left(u''(\mu + \sigma x) \int_a^x x dF(x) \right) dx}{\int_a^b u'(\sigma + \mu x) dF(x)} \quad (4)$$

- *Property 2:* $V_{\sigma}(\sigma, \mu) < 0$ for all μ and all $\sigma > 0$ if and only if $u''(\alpha + \beta x) \leq 0$ for all $\alpha + \beta x$.

- *Property 3:* $S(\sigma, \mu) \geq 0$ for all μ and all $\sigma > 0$ if and only if $u'(\alpha + \beta x) > 0$ and $u''(\alpha + \beta x) < 0$ for all $\alpha + \beta x$.

$$\begin{aligned} V_{\mu\mu}(\sigma, \mu) &= \int_a^b u''(\mu + \sigma x) dF(x) \\ V_{\mu\sigma}(\sigma, \mu) &= \int_a^b u''(\mu + \sigma x) x dF(x) \\ V_{\sigma\sigma}(\sigma, \mu) &= \int_a^b u''(\mu + \sigma x) x^2 dF(x) \end{aligned} \quad (5)$$

- *Property 4:* $V(\sigma, \mu)$ is a concave function for all μ and all $\sigma > 0$ if and only if $u''(\alpha + \beta x) \leq 0$ for all $\alpha + \beta x$.
- *Property 5:* $dS(\sigma, \mu)/d\mu < (=, >) 0$ for μ and all $\sigma > 0$ if and only if $u(\alpha + \beta x)$ displays decreasing (constant, increasing) absolute risk aversion for all $\alpha + \beta x$.

- *Property 6:* $dS(t\sigma, t\mu)/dt < (=, >) 0$ for μ and all $\sigma > 0$ if and only if $u(\alpha + \beta x)$ displays decreasing (constant, increasing) absolute risk aversion for all $\alpha + \beta x$.
- *Property 7:* $S_1(\sigma, \mu) \geq S_2(\sigma, \mu)$ for all (σ, μ) if and only if $u_1(\alpha + \beta x)$ is more risk averse than $u_2(\alpha + \beta x)$.