

Von Neumann-Morgenstern: Lecture XI

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- 1 Numerical Stuff
- 2 von Neumann and Morgenstern
 - Axioms
 - Ordering and Combining

Numerical Stuff

- In the preceding lecture, we found the expected utility of a gamble that paid \$150,000 with probability of 0.6 and \$50,000 with probability 0.4. Assuming a $r = 0.5$, the power utility function yields a certainty equivalent of \$103,569.
- Let's work on a slightly different problem, again assume that we have a risky gamble that pays \$150,000 with some probability p and \$50,000 with probability $(1 - p)$.
 - I assert that we can find a p that makes the decision maker indifferent between the risky gamble and a certain payoff of \$108,000. Naturally, we assume that p is higher than 0.6 (why?).
 - Using our power utility function, we know that

$$U(108,000) = \frac{108,000^{0.5}}{0.5} = 657.27 \quad (1)$$

- Continued

- Mathematically, the probability then becomes

$$\begin{aligned}657.27 &= 774.60p + 447.21(1 - p) \\657.27 &= 774.60p + 447.21 - 447.21p \\210.05 &= 327.38p \\p &= 0.6416\end{aligned}\tag{2}$$

- Changing the problem slightly, assume that the payoffs are \$150,000 with probability 0.6 and \$50,000 with probability 0.4. What is the r required to make the certainty equivalent \$108,000?
 - Our conjecture is that this risk aversion is less than the original risk aversion of .5 (why?).
 - Borrowing from the above analysis, the problem this time is slightly different

$$\frac{108,000^{1-r}}{1-r} = 0.6 \frac{150,000^{1-r}}{1-r} + 0.4 \frac{50,000^{1-r}}{1-r} \quad (3)$$

- As long as r does not equal 1, we can simplify the problem and write it in implicit functional form as

$$g(r) = 108,000^{1-r} - 0.6 \cdot 150,000^{1-r} - 0.4 \cdot 50,000^{1-r} = 0 \quad (4)$$

- The conjectures under Von Neumann and Morgenstern are actually close to the first problem.
 - Assume that you have three points A , B , and C . Further, assume that points B and C represent a risky gamble with probabilities 0.50, 0.50.
 - The producer can tell you whether he prefers the risk-free point A and the risky gamble B/C .
 - Rule out the cases where A is preferred to both B and C and B and C are preferred to A .

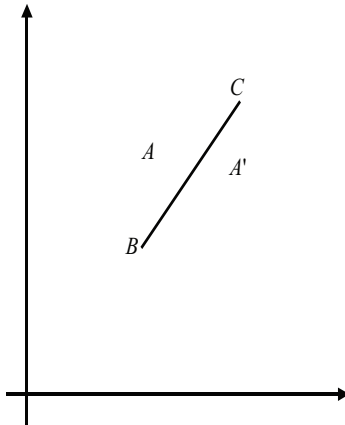


Figure: Utility Ordering

- Continued

- Mathematically, either A is preferred to the gamble or lottery between B and C

$$A \succ B \circ C \quad (5)$$

or the lottery is preferred to A

$$B \circ C \succ A \quad (6)$$

- As a second postulate, as depicted in the previous example, we can define a probability that makes the lottery indifferent with the certain payoff.

- Conceptual Structure of the Axiomatic Treatment of Numerical Utilities
 - In an axiomatic treatment, we want to propose a set of axioms or basic notions that are acceptable and show that a conclusion follows from direct logic based on these axioms or notions.
 - In this case, we want to show that there exists a utility mapping $U(Y)$ such that if X is preferred to Z then $U(X) > U(Z)$.
 - Axioms
 - $u \succ v$ is a complete ordering of U : For any two u, v one and only one of the three following relations hold

$$u \approx v \quad u \succ v \quad u \prec v \quad (7)$$

- $u \succ v$ and $v \succ w$ implies that $u \succ w$: Basically the axiom assumes that preferences are transitive.

- Ordering and Combining

- $u \prec v$ implies that $u \prec \alpha u + (1 - \alpha) v$
- $u \succ v$ implies that $u \succ \alpha u + (1 - \alpha) v$
- $u \prec w \prec v$ implies the existence of α such that $\alpha u + (1 - \alpha) \prec w$
- $u \succ w \succ v$ implies the existence of α such that $\alpha u + (1 - \alpha) \succ w$.