Lecture XXVI: Decisions Under Crop Insurance

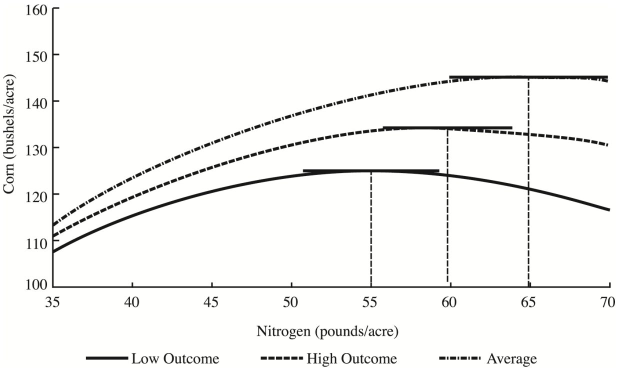
Charles B. Moss [1]

[1] University of Florida

November 13, 2018

Effect of Insurance on Profit Maximization

- As a starting point, assume that we have three related production function: (1) Production with High Rainfall, (2) Production with Low Rainfall, and (3) Average Production.



State-Dependent Production Functions

- The production function used to choose controllable inputs (Nitrogen) becomes

$$f(x) = \begin{cases} f_1(x) = 4.5097x - 0.0407x^2 \text{ if } \omega = \omega_1 \text{ and } \mathrm{P}[\omega_1] = 0.50 \\ f_2(x) = 4.4258x - 0.0338x^2 \text{ if } \omega = \omega_2 \text{ and } \mathrm{P}[\omega_2] = 0.05 \end{cases} \tag{1}$$

- Following standard assumptions, the farmer chooses the level of variable input that maximizes expected profits

$$\begin{aligned} \pi = \max_x 0.50 \left[\left(4.5097x - 0.0407x^2\right) p_y - p_x x\right] + \\ 0.50 \left[\left(4.4258x - 0.0338x^2\right) p_y - p_x x\right] \end{aligned} \tag{2}$$

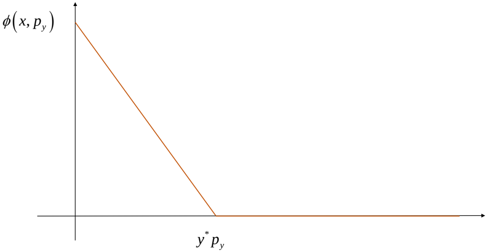
- This can be reformulated as maximize profit given the average production function

$$\pi = \max_x \left[4.4678x - 0.0372x^2\right] 3.50 - 0.125x \tag{3}$$

Optimal Input Level – Average Production Function

- The result is $x^* = 59.536$ and $\pi^* = 461.77$.
- Next, consider the payoff of an insurance policy

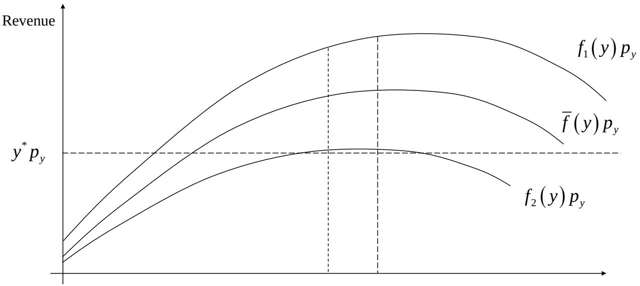
$$\phi(x,\pi) = \max\left[y^* - f(x), 0\right] p_y = \begin{cases} \left(y^* - f(x)\right) p_y \text{ if } f(x) < y^* \\ 0 \text{ if } f(x) \geq y^* \end{cases} \tag{4}$$



Revenue Under Insurance

- Putting Market Revenue with the gain to insurance

$$R(y^*, x, p_y) = \max\left[y^*, f(x)\right] p_y = \begin{cases} y^* p_y \text{ if } f(x) \leq y^* \\ f(x)\, p_y + 0 = f(x)\, p_y \text{ if } f(x) > y^* \end{cases} \quad (5)$$



Profit with Insurance

- Equation 5 is the same as the insurance payoff plus the revenue from production

$$f(x)\,p_y + \phi(x,p_y) = \begin{cases} f(x)\,p_y + (y^* + f(x))\,p_y = y^* p_y \text{ if } f(x) < y^* \\ f(x)\,p_y + 0 = f(x)\,p_y \text{ if } f(x) > y^* \end{cases} \tag{6}$$

- The firm's profit function then becomes

$$\begin{aligned} \pi = \max_x 0.5 &\left[\max\left[4.5097x - 0.0407x^2, 125\right] 3.50 - 0.125x\right] + \\ 0.5 &\left[\max\left[4.4258x - 0.0338x^2, 125\right] 3.50 - 0.125x\right] - \psi \end{aligned} \tag{7}$$

where ψ is the price of insurance.

- If the price of insurance is constant, the optimum level of nitrogen becomes $x^* = 64.471$ and the profit before insurance increases to 653.21 .

Pricing Insurance

- The actuarial price of insurance can be expressed as

$$\psi^*\left(\theta,x,y^*\right)=p_y\left[\theta\max\left[y^*-f_1\left(x\right),0\right]+\left(1-\theta\right)\max\left[y^*-f_2\left(x\right),0\right]\right] \tag{8}$$

  where θ is the probability of a high rainfall event.

- If we substitute the actuarially fair price of insurance into Equation 7

$$\begin{aligned}\pi=&\max_x 0.5\left[\max\left[4.5097x-0.0407x^2,125\right]3.50-0.125x\right]+\\ &0.5\left[\max\left[4.4258x-0.0338x^2,125\right]3.50-0.125x\right]-\psi\left(\theta,x,y^*\right)\end{aligned} \tag{9}$$

  which yields the original solution (i.e., $x^*=59.536$ and $\pi^*=461.77$).

- However, the risk declines – there is less risk with insurance.

Pricing Insurance – Subsidy and Load

- Next, consider the subsidy paid on crop insurance policies in the United States

$$\pi = \max_x 0.5\left[\max\left[4.5097x - 0.0407x^2, 125\right]3.50 - 0.125x\right] + 0.5\left[\max\left[4.4258x - 0.0338x^2, 125\right]3.50 - 0.125x\right] - (1-s)\,\psi(\theta, x, y^*) \tag{10}$$

  where s is the percent subsidy.
- Intuitively, if $s > 0$ the optimal level of fertilizer increases above the original solution – the level or risk increases.
- We could also consider the cost of the load (l)

$$\pi = \max_x 0.5\left[\max\left[4.5097x - 0.0407x^2, 125\right]3.50 - 0.125x\right] + 0.5\left[\max\left[4.4258x - 0.0338x^2, 125\right]3.50 - 0.125x\right] - (1-s)(1+l))\psi(\theta, x, y^*) \tag{11}$$

Supply of Crop Insurance

- The profit function for the insurance company can be written as

$$\pi = r_t \left[K_t + \psi^* \left(\theta_t, x_t, y_t^*\right)\right] + \chi \left[\Psi^* \left(\theta_t, x_t, y_t^*\right), C\left(\theta_t - \theta_t^*, z_t\right)\right] \tag{12}$$

  where
  - r_t is the return on capital.
  - K_t is the companies capital reserve.
  - $\chi\left(\Psi, C_t\right)$ is the net profit to underwriting insurance.
  - C_t is the cost of operations
- The problem with the theoretical profit function in Equation 12 is the assumption that we can observe the true riskiness of production (θ).
- In the current formulation, we assume that the insurance company must expend resources (underwriting expenses) to determine the true value of θ.

Underwriting

- If we assume the true value of θ to be θ^*, any estimated value of θ less than θ^* (an undervaluation of the probability of a low yield event) will produce a return per unit of insurance sold less than zero

$$E\left[\Psi\left(\theta, x, y^*\right) - \Psi\left(\theta^*, x, y^*\right)\right] < 0 \text{ if } \theta < \theta^* \tag{13}$$

- If the insurance company sets the probability of a low outcome event too high, the number of policies sold will decline (i.e., only increasing risk averse individuals will buy insurance at higher premiums).

Underwriting, Continued

- The return can be rewritten as

$$\begin{aligned}\chi\left(\Psi^*\left(\theta_t, x_t, y_t^*\right), C\left(\theta_t, z_t\right)\right) = \mathrm{E}\left[\Psi\left(\theta_t, x_t, y_t^*\right) - \Psi\left(\theta_t^*, x_t, y_t^*\right)\right] \times \\ q_t\left(\Psi\left(\theta_t, x_t, y_t^*\right)\right) - C\left(\theta_t - \theta_t^*, z_t\right)\end{aligned} \tag{14}$$

- Loss ratio

$$\lambda = \frac{\Psi\left(\theta^*, x, y^*\right)}{\Psi\left(\theta, x, y^*\right)} = \frac{\mathrm{E}\left[\max\left(y^* - \left[b\left(\omega\right)x + c\left(\omega\right)x^2\right], 0\right) p_y \middle| \theta^*\right]}{\Psi\left(\theta, x, y^*\right)} \tag{15}$$

Reinsurance

- As a starting point, let us develop the return to insurer.

Farmer	Farmer Yield	Ins. Yield Paid	Insurance Indemnity
1	125.00	0.77	-1.35
2	143.69	0.00	1.35
3	143.69	0.00	1.35
4	143.69	0.00	1.35
5	143.69	0.00	1.35
6	125.00	0.77	-1.35
7	125.00	0.77	-1.35
⋮	⋮	⋮	⋮
100	143.69	0.00	1.35

Income to Insurer

- The profit to this "first draw" is -16.24.
- Suppose we draw this over and over again.

Draw	Profit	Draw	Profit	Draw	Profit
1	-16.2386	11	13.5258	21	-8.1211
2	13.5258	12	-2.7094	22	-13.5328
3	-0.0035	13	-0.0035	23	5.4082
4	-5.4152	14	16.2316	24	-18.9445
5	-24.3562	15	5.4082	25	13.5258
6	8.1141	16	10.8199	26	13.5258
7	2.7024	17	13.5258	27	-0.0035
8	10.8199	18	-2.7094	28	21.6433
9	-10.8269	19	-2.7094	$\vdots$	$\vdots$
10	-18.9445	20	-8.1211	200	5.4082

Summary of Insurance Returns

Value	Estimate
Minimum	-35.180
1st Quantile	-8.121
Median	1.349
Mean	1.385
Std. Dev	12.659
3rd Quantile	10.820
Maximum	29.761

Insuring the Insurer

- Reinsurance is basically insuring the insurer.
- Expected Utility of HARA Utility function

$$\begin{aligned} CE &= (1-x)\,\mu - \frac{\rho\,(1-x)^2\,\sigma^2}{2} \\ &= (1-x)\,1.3825 - \frac{\rho\,(1-x)^2\,160.2426}{2} \end{aligned} \tag{16}$$

Optimum Reinsurance

	ρ	x	CE
1	0.0500	0.82745	0.11928
2	0.0450	0.80828	0.13253
3	0.0400	0.78431	0.14909
4	0.0350	0.75350	0.17039
5	0.0300	0.71242	0.19879
6	0.0250	0.65490	0.23855
7	0.0200	0.56862	0.29819
8	0.0150	0.42483	0.39759
9	0.0125	0.30980	0.47710
10	0.0100	0.13725	0.59638

Reinsurance and Correlation

- One of the most significant factors pushing crop insurance firms toward reinsurance is potential correlation among farms in a small regional area.
- First, assume that the insurance claims for the firms are uncorrelated

$$\begin{aligned} \mathrm{E}\left[\sum_{i=1}^{n} x_i\right] &= n\mu_i \\ \mathrm{V}\left[\sum_{i=1}^{n} x_i\right] &= n\sigma_i^2 \end{aligned} \tag{17}$$

Reinsurance and Correlation, Continued

- Assume now that the insurance claims for the firms are correlated

$$\begin{aligned} \mathrm{V}\left[\sum_{i=1}^{n} x_i\right] &= \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \sigma_{ij} \\ &= n\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \rho_{ij}\sigma_i\sigma_j \end{aligned} \tag{18}$$

- So if insurance payments are positively correlated between two farms (i.e., $\rho_{ij} > 0$), the risk of writing crop insurance increases.

Risk Management Agency – Standard Reinsurance Contract

- The Company, in accordance with its Plan of Operations, may designate an eligible crop insurance contract to the Assigned Risk Fund by State.
- Any eligible crop insurance contract not specifically designated by the Company to the Assigned Risk Fund will automatically be assigned to the Commercial Fund by State.

Assigned Risk Fund Retention

- The Company shall retain a 20 percent interest in premium and associated ultimate net losses in the Assigned Risk Fund in each State. The remainder is ceded to FCIC.
- The associated net book premium of eligible crop insurance contracts assigned to the Assigned Risk Fund shall not exceed 75 percent of the Company's net book premium in each State.

Commercial Retention Fund

- The Company shall retain at least a 35 percent interest in premium and associated ultimate net losses in the Commercial Fund in each State. The remainder shall be ceded to FCIC.
- The retention percentage for the Commercial Fund in each State shall be made in 5 percent increments and designated in the Company's Plan of Operations ....

Underwriting Loss – Commercial Fund

- For that portion of the underwriting loss amount for which the Company's loss ratio exceeds 100 percent and is less than or equal to 160 percent, the Company shall retain an amount of the underwriting loss equal to the product of the following:
  - Its retained net book premium;
  - The lesser of the Company's actual loss ratio or 160 percent, minus 100 percent; and
  - The following percentage for the applicable State Group:
    - State Group 1 – 65.0 percent
    - State Group 2 and 3 – 42.5 percent

State Groups

- **State Group 1** means Illinois, Indiana, Iowa, Minnesota, and Nebraska.
- **State Group 2"** means Alabama, Arizona, Arkansas, California, Colorado, Florida, Georgia, Idaho, Kansas, Kentucky, Louisiana, Michigan, Missouri, Mississippi, Montana, North Carolina, North Dakota, New Mexico, Ohio, Oklahoma, Oregon, South Carolina, South Dakota, Tennessee, Texas, Virginia, Washington, and Wisconsin.
- **State Group 3** means Alaska, Connecticut, Delaware, Hawaii, Maine, Massachusetts, Maryland, Nevada, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Utah, Vermont, West Virginia, and Wyoming.

Underwriting Loss – Commercial Fund, Continued

- For that portion of the underwriting loss amount for which the Company's loss ratio exceeds 160 percent and is less than or equal to 220 percent, the Company shall retain an amount of the underwriting loss equal to the product of the following:
  - Its retained net book premium;
  - The lesser of the Company's actual loss ratio or 220 percent, minus 160 percent; and
  - The following percentage for the applicable State Group:
    - State Group 1 – 45.0 percent
    - State Group 2 and 3 – 20.0 percent

Underwriting Loss – Assigned Risk Fund

- For that portion of the underwriting loss amount for which the Company's loss ratio exceeds 100 percent and is less than or equal to 160 percent, the Company shall retain an amount of the underwriting loss equal to the product of the following:
  - Its retained net book premium;
  - The lesser of the Company's actual loss ratio or 160 percent, minus 100 percent; and
  - 7.5 percent.

Underwriting Gain – Commercial Fund

- For that portion of the underwriting gain amount for which the Company's loss ratio is less than or equal to 100 percent but is greater than or equal to 65 percent, the Company shall retain an amount of the underwriting gain equal to the product of the following:
  - Its retained net book premium;
  - 100 percent minus [the greater of the Company's actual loss ratio or 65 percent]; and
  - The following percentage for the applicable State Group:
    - State Group 1 – 75.0 percent
    - State Group 2 and 3 – 97.5 percent