Lecture XXIV: Changing to Risk Based Agricultural Policies

Charles B. Moss [1]

[1] University of Florida

November 5, 2018

A Little Calculus

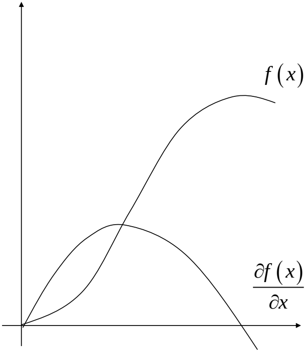
- While Calculus is not required, I hope you have seen a little of it.
- Most of familiar with differentiation

$$\frac{\partial x^n}{\partial x} = n x^{n-1} \tag{1}$$

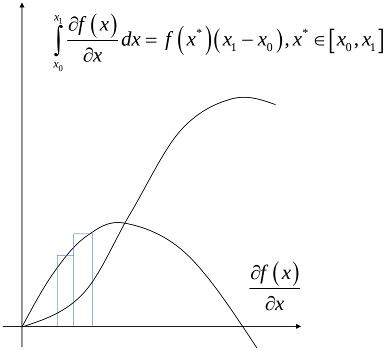
I need to develop the concept of integration.

- Differentiation is determining the slope of a curve at any point.
- Integration is determining the area underneath the curve.

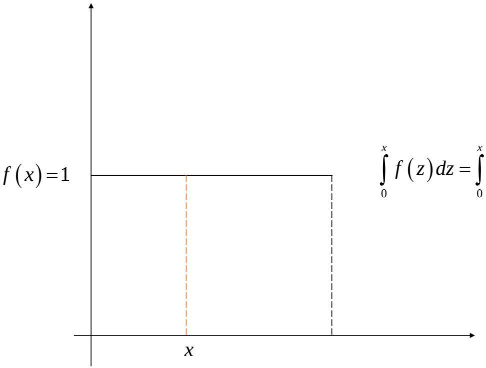
Differentiation and Integration



Production Function

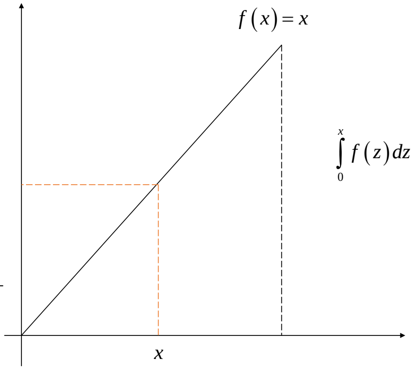


Integrating a Constant Function



$$\int_0^x f(z)dz = \int_0^x 1dz = x + k$$

Integrating a Linear Function

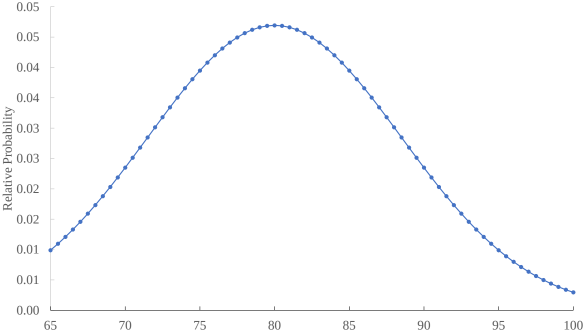


$$\int_0^x f(z)\,dz = \int_0^x z\,dz = \frac{x^2}{2} + k$$

$$\frac{(x-0)(x-0)}{2} = \frac{x^2}{2}$$

Normal Distribution

- What I want to discuss is the scenario where the function is a probability density function - the relative probability that one event occurs.
- For the normal distribution



Normal Distribution – Payoff

- In this example, the relative probability of a yield of 75 would be 0.039478. The relative probability of a yield of 80 would be 0.46934.
- What I want to develop is the value of program that "payoffs" in certain states of nature.
- The Agricultural Risk Coverage (ARC) program pays off if the total revenue falls below a certain level.

$$\text{Payoff} = 0.80 \times (\tilde{p} \times \tilde{y}) - p \times y - \quad (2)$$

- Let us suppose that $\tilde{p}$ is 3.40/bu. and $\tilde{y}$ is 85 - $0.80 \times (\tilde{p} \times \tilde{y}) = 231.20$.
  - Suppose the yield fell to 75 and the price fell to 3.00/bu - the ARC would payout 6.20/acre.
  - However, if the yield was 80 and the price fell to 3.00/bu - there would be no payout ($ 240.0/acre > $ 231.20/acre).

Simple ARC Model

- However, based on normal distribution function - a yield of 80 bu. (and a payoff of zero) is more likely than a yield of 75 bu. (and a payoff of 6.20/acre)
- In this framework, it makes sense to talk about the expected value of program

$$\mathrm{E}\left[\mathrm{Payoff}\right] = \sum_{i=1}^{N} \mathrm{P}\left[y_i\right] \max\left(231.20 - 3.00 \times y_i, 0\right) \tag{3}$$

where y_i is some level of yield and $\mathrm{P}\left[y_i\right]$ is the probability of that yield.

Probability Density Function - ARC

- Replacing the discrete random variable in Equation 3 with a continuous random variable

$$E\left[\text{Payoff}\right] = \int_{-\infty}^{\infty} f\left(y\right) \max\left(231.20 - 3.00y, 0\right) dy \quad (4)$$

- Let's consider a simple form of the uniform distribution function

$$f\left[y\right] = \frac{1}{90 - 55} \text{ for } y \in \left[55, 90\right]. \quad (5)$$

- Hence

$$E\left[\text{Payoff}\right] = \frac{1}{35} \int_{55}^{90} \max\left(231.20 - 3.00 \times y, 0\right) dy \quad (6)$$

Probability Density Function - ARC, Continued

- Next, we solve for the value of y so that

$$231.20 - 3.00 \times y = 0 \Rightarrow y = \frac{231.20}{3.00} = 77.07bu. \quad (7)$$

- If the yield is 77.07 bu. at 3.00/bu - the gross revenue is 321.20.
- At this yield $\max\left(231.20 - 3.00 \times y, 0\right) = 0$. Hence, the integral in Equation 6 becomes

$$\begin{aligned} \mathrm{E}\left[\text{Payoff}\right] &= \frac{1}{35}\int_{55}^{77.07}\left(231.20 - 3 \times y\right)dy \\ &= \frac{231.20}{35}\int_{55}^{77.07}dy - \frac{3}{35}\int_{55}^{77.07}ydy \end{aligned} \quad (8)$$

More Calculus

- Taking the first part

$$6.6057 \int_{55}^{77.07} dy = 6.6057 \left(y|_{55}^{77.07} \right. \\ = 6.6057 \left(77.07 - 55\right) = 145.77 \tag{9}$$

- Taking the second part

$$0.0857 \int_{55}^{77.07} y dy = 0.0857 \times \frac{1}{2} \left(y^2|_{55}^{77.07} \right. \\ = 0.0486 \left(5,939.27 - 3025.00\right) = 124.903 \tag{10}$$

- Expected Payoff - $145.77 - 124.903 = 20.867$.

Integration by Simulation

- This is complicated enough with a simple distribution function. Consider integrating the normal density function

$$f\left(x|\,\mu,\sigma^2\right)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{\left(x-\mu\right)^2}{\sigma^2}\right) \qquad (11)$$

- Actually, this function does not have a closed form integral.
- The alternative that has become popular is to integrate by simulation.
- Computer code can be used to draw random variables from our distribution.

Integration by Simulation

z_1	Payoff z_1	z_2	Payoff z_2
76.880	0.560	70.702	19.094
89.733	0.000	76.696	1.112
89.994	0.000	62.081	44.958
67.337	29.190	51.163	77.711
74.363	8.112	67.931	27.407
59.534	52.597	81.082	0.000
86.951	0.000	65.821	33.736
55.696	64.113	72.771	12.886
63.651	40.248	92.710	0.000
77.965	0.000	75.040	6.079
	19.4382		22.298

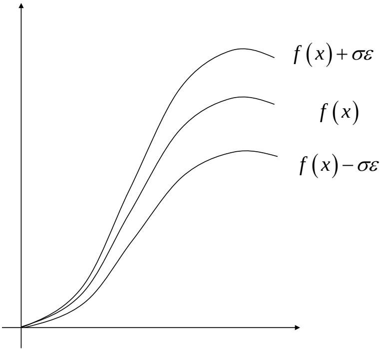
Section 1412.53 ARC Payment Provisions

- Payment is equal to the result of multiplying the payment acres for the covered commodity times the difference between the actual crop revenue and the ARC-CO guarantee, not to exceed 10 percent of the ARC-CO benchmark revenue.
  - ARC-IC – Average Revenue Coverage - Individual coverage
  - ARC-CO – Average Revenue Coverage - County Coverage
- "ARC-CO makes payments when actual county crop revenue is below 86% of county benchmark revenue. Specifically, ARC-CO payment/program acre = minimum [(86% times ARC revenue benchmark) – ARC actual revenue; 10% times ARC revenue benchmark]" – Zulauf "Agricultural Risk Coverage – County (ARC-CO) vs. Price Loss Coverage"

Real Corn Data for Florida

Year	Price	Yield	Olympic Price
2009	4.00	97	
2010	4.70	109	
2011	6.65	104	
2012	7.50	115	
2013	4.51	133	
2014	3.65	135	
2015	3.80	141	3.80
2016	3.93	145	3.93
2017	4.15	161	4.15
Avg	4.77	126.67	3.96

- Assuming a Program Yield of 115.0
- ARC Guarantee Revenue 391.64
- Max ARC Payoff 39.16



Production Function