Lecture XIV: Review of Economic Equilibrium and Consumer Surplus

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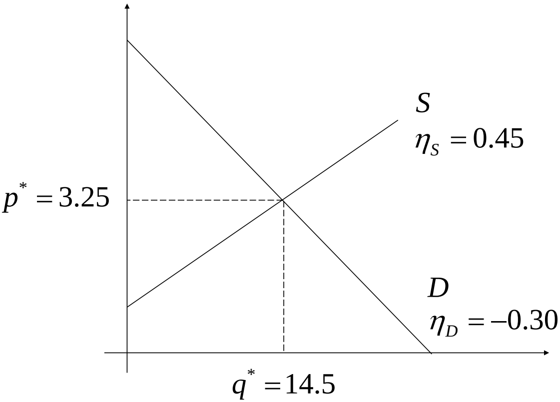
1. Economic Equilibrium
2. Consumer Surplus
3. More Advanced Demand Curve

Example 1 – Graph



Example 1 – Linear Supply

- Let us start with linear supply and demand relationships.
- The supply function can be written as

$$Q_s = a + bp \tag{1}$$

- To determine b, we use the elasticity

$$\begin{aligned} \eta_S = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} \\ b = \eta_S \frac{Q}{P} \end{aligned} \tag{2}$$

Example 1 – Linear Supply, Continued

- Therefore, we can use the elasticity of supply

$$b = \frac{\Delta Q_S}{\Delta P} = 0.45 \frac{14.5}{3.25} = 2.0077 \quad . \tag{3}$$

- To compute the constant

$$\begin{array}{c} a = 14.5 - 2.0077 \cdot 3.25 = 7.9750 \\ Q_S = 7.9750 + 2.0077P \\ Q_S = 7.9750 + 2.0077 \times 3.25 = 14.5 \end{array} \quad . \tag{4}$$

Example 1 - Inverted Supply Curve

- Imagine working the problem for price instead of quantity

$$P = \tilde{a} + \tilde{b} Q_S \\ b = \frac{\Delta P}{\Delta Q} \tag{5}$$

- Applying Green's Theorem

$$\frac{\Delta P}{\Delta Q_S} = \frac{1}{\frac{\Delta Q_S}{\Delta P}} = \frac{1}{\eta_S} \frac{P}{Q} = \frac{1}{0.45} \frac{3.25}{14.5} = 0.4981 \tag{6}$$

- Solving for the constant

$$\tilde{a} = 3.25 - 0.4981 \times 14.5 = -3.9722 \\ P = -3.9772 + 0.4981 \times 14.5 = 3.25 \tag{7}$$

Example 1 – Linear Demand

- Same with the Demand curve

$$\eta_D = -0.30 \Rightarrow \frac{\Delta Q_D}{\Delta P} = -0.30 \times \frac{14.5}{3.25} = -1.3385. \qquad (8)$$

- For the constant

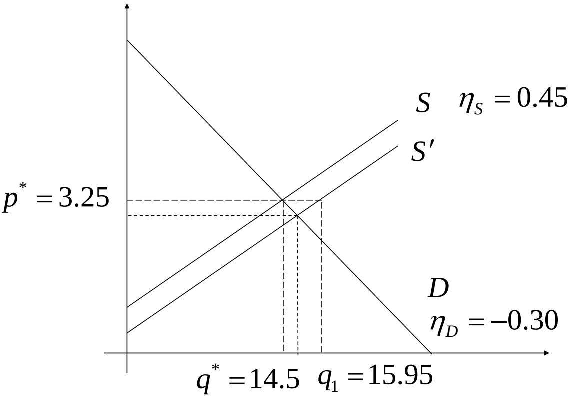
$$\begin{aligned} a &= 14.5 + 1.3385 \cdot 3.25 = 18.85 \\ Q_D &= 18.85 - 1.3385P \\ Q_D &= 18.85 - 1.3385 \times 3.25 = 14.5 \end{aligned} \qquad (9)$$

Example 1 – Inverse Demand Curve

- As with the supply curve – we can invert the demand curve

$$\begin{aligned} P &= \tilde{a} + \tilde{b} Q_D \\ \tilde{b} = \frac{\Delta P}{\Delta Q_D} &= \frac{1}{\eta_D} \frac{P}{Q_D} = \frac{1}{-0.30} \frac{3.25}{14.5} = -0.7471 \\ \tilde{a} &= 3.25 + 0.7471 \times 14.5 = 14.0830 \\ P &= 14.0830 - 0.7471 \times 14.5 = 3.25 \end{aligned} \tag{10}$$

Example 1 – 10 % Increase in Supply



Example 1 – New Supply Curve

- Shift in Supply

$$14.5 \times 1.10 = 15.95 \tag{11}$$

- New Supply Curve

$$\begin{aligned} a &= 15.95 - 2.0077 \times 3.25 = 9.4250 \\ Q'_S &= 9.4250 + 2.0077 \times 3.25 = 15.95 \end{aligned} \tag{12}$$

- Solve for the new Equilibrium

$$\begin{aligned} 9.4250 + 2.0077P &= 18.85 - 1.3385P \\ (2.0077 + 1.3385)\,P &= 18.85 - 9.4250 \\ 3.3462P &= 9.4250 \\ P &= 2.8167 \end{aligned} \tag{13}$$

Example 1 – New Quantity

- New Quantity

$$\begin{aligned} 9.4250 + 2.0077 \times 2.8167 = 15.08 \\ 18.85 - 1.3385 \times 2.8167 = 15.08 \end{aligned} \tag{14}$$

Working from the Price Side

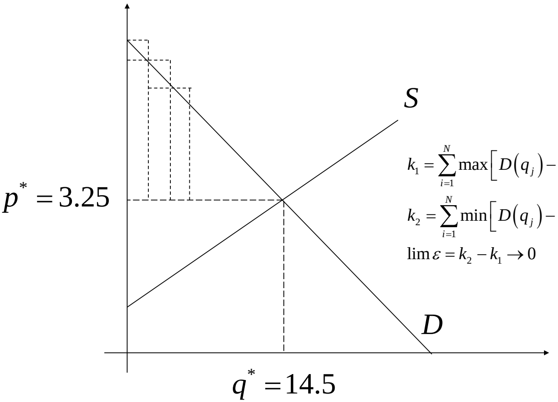
- Let us resolve the inverse supply function given the shift

$$\begin{aligned} \tilde{a} &= 3.25 - 0.4981 \times 15.95 = -4.6947 \\ P &= -4.6947 + 0.4981 \times 15.95 = 3.25 \end{aligned} \tag{15}$$

- Now instead of equating the quantities, we equate the prices

$$\begin{aligned} -4.6947 + 0.4981Q &= 14.0830 - 0.7471Q \\ (0.4981 + 0.7471)\,Q &= 14.0830 + 4.6947 \\ Q &= \frac{18.7777}{1.2452} = 15.08 \end{aligned} \tag{16}$$

Consumer Surplus



$$k_1 = \sum_{i=1}^{N} \max\left[D(q_j) - 3.25 \middle| p_j \in [q_i, q_{i+1}] \right] (q_{i+1} - q_i)$$

$$k_2 = \sum_{i=1}^{N} \min\left[D(q_j) - 3.25 \middle| p_j \in [q_i, q_{i+1}] \right] (q_{i+1} - q_i)$$

$$\lim \varepsilon = k_2 - k_1 \to 0$$

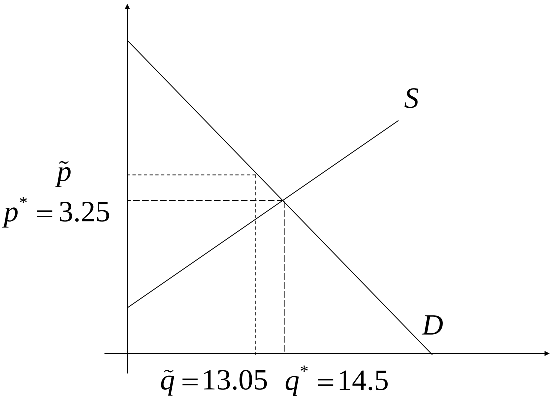
Quantifying Consumer Surplus – A Simple Approach

- Remembering your calculus, the area in Figure 3 is defined as the Reimann sum – the integral

$$CS = \int_0^{q^*} D\left[q\right] dq. \tag{17}$$

- However, this is not quite correct (Marshall drew the graphy wrong).
- There is also a simpler approach that fits most of our needs.
- Starting with the original solution from Example 1 – consider a quota on production.
- Suppose that there is a government action that puts a limit of 90 % of ordinary production on the firms – $14.5 \times 0.90 = 13.05$.

Quantifying Consumer Surplus – Quota



Quantifying Consumer Surplus – Quota, Continued

- What is the new price?

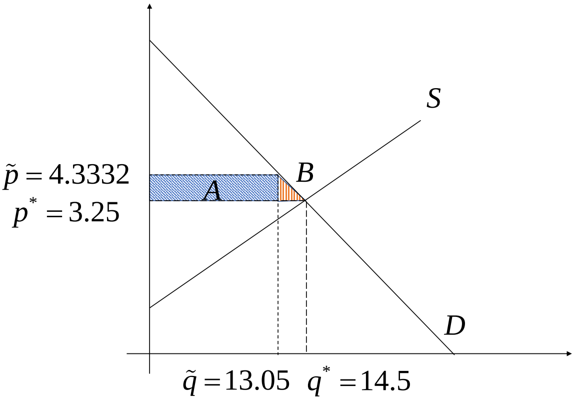
$$\begin{aligned} 13.05 &= 18.85 - 1.3385p \\ 13.05 - 18.85 &= -1.3385p \\ \frac{-5.80}{-1.3385} &= p = 4.3332 \end{aligned} \tag{18}$$

- Checking the solution

$$D = 18.85 - 1.3385 \times 4.3332 = 13.05 \tag{19}$$

- A little analytical geometry

Quantifying Consumer Surplus – Quota, Continued



Quantifying Consumer Surplus – Quota, Continued

- We can compute these areas with a little geometry

$$\begin{aligned} A &= (4.3332 - 3.25) \times 13.05 = 14.1358 \\ B &= \frac{1}{2}(4.3332 - 3.25) \times (14.5 - 13.05) = 0.7853 \end{aligned} \quad (20)$$

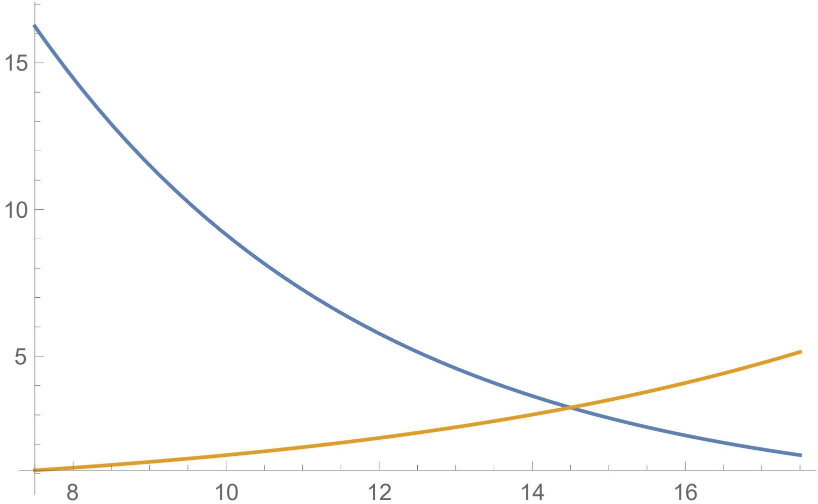
- $\Delta CS = 14.1358 + 0.78532 = 14.9211$

More Advanced Demand Curve

- The simple supply and demand curves above can be viewed as first-order Taylor expansions of more general supply and demand curves

$$\begin{aligned} p &= 0.3522 \exp\left(0.1533 q_s\right) \\ p &= 91.1028 \exp\left(-0.2299 q_d\right) \end{aligned} \tag{21}$$

Exponential Supply and Demand Curve

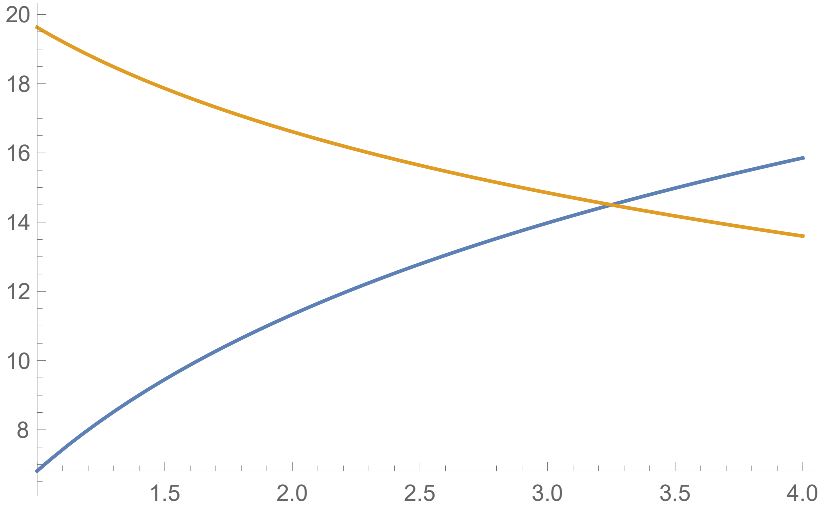


Solving for Price in Terms of Quantity

- Solving for quantity in terms of price yields

$$
\begin{aligned}
q_S &= 6.525 \ln \left(2.8393p\right) \\
q_d &= -4.35 \ln \left(0.01098p\right)
\end{aligned} \tag{22}
$$

Graph of Price Dependent Quantities



A Real Measure of Consumer Surplus

- Returning to the integral definition

$$\begin{aligned} \Delta CS = -\int_{3.25}^{4.3332} & -4.35\ln\left(0.01098p\right)dp \\ & = 14.9963 \end{aligned} \quad (23)$$