Lecture XXVI: Debt Choice

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1. Overview of Debt Choice
   - Reinvesting Income
   - Debt Choice Under Certainty
   - Debt Choice Under Risk

Overview of Debt Choice

- Meyers (1984) references the Modigliani-Miller theorem (Modiglianni and Miller, 1958) as a starting point for discussions of capital structure.
  - Modigliani-Miller suggests that the value of an asset should be independent of choice of capital structure (i.e., debt-to-asset ratio).
  - Meyers concludes that the observed debt-to-asset positions should be fairly close for similar firms if the transaction cost of adjusting the capital structure is small.
  - The divergence in capital structure suggests that the transaction cost is large enough to sustain capital structures out of equilibrium.

- In agriculture, the selection of capital structure leads to two different questions.
  - First, why is the capital market in agriculture dominated by the debt market?
  - Second, given that most of the external equity for agriculture will be in the form of debt, what determines the firm's choice of debt level?
- Consider the R_E, the return on equity (not the rate of return on equity, but the return on equity) written as

$$R_E = r_A \left[E \left(1 + \Delta_E \right) + D \left(1 + \Delta_D \right) \right] - KD \left(1 + \Delta_D \right) \quad (1)$$

where r_A is the rate of return on assets, E is the level of equity, Δ_E is the percent change in the level of equity, D is the level of debt, Δ_D is the percent change in the level of debt, and K is the cost of capital (an interest rate).

- This expression can be changed slightly to yield

$$R_E = r_A E \left(1 + \Delta_E\right) + \left(r_A - K\right) D \left(1 + \Delta_D\right). \tag{2}$$

- The farm could either consume its net income (i.e., owner withdraws) or reinvest the returns into the farm. This concept can be written as

$$\Delta E = r_E E - C \tag{3}$$

where r_E is the rate of return on equity and C is the level of consumption.
- Rewriting this expression

$$\begin{aligned} \frac{\Delta E}{E} &= r_E - \frac{C}{E} \\ \Rightarrow \Delta_E &= r_E - \frac{C}{E} \end{aligned} \tag{4}$$

so that Δ_E is the percentage change in owner equity.

- Substituting this result into Equation 2 yields

$$R_E = r_A E \left(1 + r_E - \frac{C}{E}\right) + (r_A - K) D (1 + \Delta_D)$$

$$R_E = r_A E \left(1 + r_A - \left[K \frac{D}{A} + \frac{C}{E}\right]\right) + (r_A - K) D (1 + \Delta_D) . \tag{5}$$

Thus, the return to owner equity increases as equity is reinvested into the firm (see the discussion of the plowback ratio in Section 4.7).

- This increase is larger than the rate of return generated by increased levels of debt even when the borrowing is profitable. This result is similar to the concept of the "pecking order" developed by Meyers 1984 where corporations choose first to fund investment opportunities using internal sources.
- Equation 5 focuses primarily on the cost of capital, the corporate results developed by Meyers adds that expanding the size of the firm by the sale of equity dilutes the shares held by current stockholders.

Debt Choice Under Certainty

- The choice of debt levels under certainty is a no-choice scenario, the solution goes to one of two boundary conditions.
  - The boundary conditions are determined by lender restrictions as a part of their asset management efforts.
  - Lenders limit the amount of money borrowed to control the probability of default.
- Assume that $\bar{R}_A = \$150/\text{acre}$ with a price of farmland of $P_A = \$1,000/\text{acre}$.

- The profit from leverage (measured in δ or the debt to asset ratio) over the range $0 \leq \delta \leq 80\%$ becomes

$$\pi(\delta) = (\$150 - \$1,000K)\,\delta A$$
$$\Rightarrow \frac{\Delta\pi}{\Delta\delta} = 150 - 1000K = 0 \tag{6}$$
$$\Rightarrow \begin{cases} K < 0.15 \Rightarrow \delta = 80\% \\ K > 0.15 \Rightarrow \delta = 0\% \end{cases}$$

where K is the cost of capital.

Debt Choice Under Risk

- Starting from our DuPont expansion based on Collins 1985

$$r_E = \left[\frac{r_p}{A} + \iota - \delta K \right] \frac{1}{1-\delta} \tag{7}$$

where r_E is the rate of return on equity, r_p is the return on the farm's operating assets, A is the firm's asset level, ι is the capital appreciation rate, δ is the debt-to-asset level, and K is the cost of capital (interest rate).

- Given that the rate of return is distributed normally,

$$\begin{aligned} \bar{r}_E &= \frac{\bar{r}_A - K\delta}{1-\delta} \\ \sigma_E^2 &= \sigma_A^2 \left(\frac{1}{1-\delta} \right)^2 \end{aligned} \tag{8}$$

- The optimal level of debt can be derived using the expected utility assuming the negative exponential utility function

$$\begin{gathered} U\left[Y\right] = -\exp\left(-\rho\left(\mu\left(Y\right) - \frac{\rho}{2}\sigma^2\left(Y\right)\right)\right) \\ CE = \mu\left(Y\right) - \frac{\rho}{2}\sigma^2\left(Y\right) \end{gathered} \tag{9}$$

the solution becomes

$$\delta^* = 1 - \frac{\rho\sigma_A^2}{\bar{r}_A - K} \Rightarrow \begin{cases} \dfrac{\Delta\delta^*}{\Delta\sigma_A^2} < 0 \\ \dfrac{\Delta\delta^*}{\Delta\bar{r}_A} > 0 \\ \dfrac{\Delta\delta^*}{\Delta K} < 0 \end{cases} . \tag{10}$$