Lecture XXIII: Market Valuation of Risk

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Revealing Risk – Changes in Market Prices

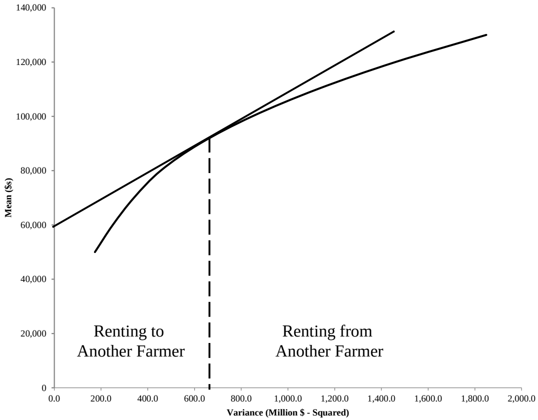
- The **revealed preference approach** typically refers to economic concepts that we can observe because the actions of individuals reveal them.
- In the case of capital markets, the fact that asset prices are lower for riskier assets reveals that investors are risk averse on average.
- In this course we are going to focus on two models of valuing risk using observed prices.
- The **Capital Asset Pricing Model** (CAPM) which has been applied to stock market data to analyze the relative riskiness of stocks.
  - The CAPM assumes that investors have information about the relative riskiness of a particular common stock and bid that information into the stock price which adjusts the rate of return on each stock.

- The **Black-Scholes Model** which is used to price options.
  - In the option market a farmer could purchase the right to sell a commodity at a specified price (a **put option**).
  - The call option gives the holder the right but not the obligation to purchase an asset such as a stock at a fixed price.
  - An insurance contract is actually similar to an option (and sometimes referred to as a **real option**) because it enables the purchaser to collect an indemnity.

Capital Asset Pricing Model

- The Capital Asset Pricing Model is derived from portfolio theory.
- We assume that investors choose the portfolio of stocks that maximize their expected utilitytaking into account the mean return and variance-covariance relationships between stock returns.
- We assume that a risk-free rate of return exists (historically government bonds) which reduces the set of all efficient investment alternatives to a single investment alternative - the market portfolio.

Expected Value – Variance with a Risk-Free Asset



- The argument is then that each stock's value must be adjusted to yield an equilibrium rate of return based on that stock's relative risk.
- The relationship between changes in the rate of return on each stock and changes in the rate of return in the market portfolio has implications for the relative risk of that stock.
- Moss 2010 gives the basic CAPM equilibrium condition as

$$E\left[R_i\right] = R_f + \beta_{im}\left(E\left[R_m\right] - R_f\right)$$
$$\beta_{im} = \frac{Cov\left[R_i, R_m\right]}{Var\left[R_m\right]} \qquad (1)$$

  - $E\left[R_i\right]$ and $E\left[R_m\right]$ are the expected rate of return for stock
  - i and the market portfolio respectively,
  - R_f is the risk-free rate of return, and
  - β_{im} is the market "beta" which represents the relative riskiness of stock i.

- There are two fundamental models for testing the CAPM using empirical data.
  - The first expression involves the estimation of β_{im} using ordinary least squares

$$R_{jt} = a_j + b_j R_{mt} + \epsilon_{jt} \tag{2}$$

    - R_{jt} and R_{mt} are the returns for stock j and the market portfolio at time t,
    - a_j is an estimated coefficient which approximates the risk-free rate of return and
    - b_j is an estimate of β_{jm}.
  - Second, that differences in relative risk – the β – explain differences in expected returns

$$\bar{R}_j = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{b}_j + u_j \tag{3}$$

    - $\bar{R}_j$ is the average rate of return on stock j and
    - $\hat{b}_j$ is the estimated β_{jm} from Equation 2.

Investment Analysis with CAPM

- Apart from testing for market equilibrium the results of the CAPM can be used to adjust the present value analysis.
- First, if we let $\tilde{P}_e$ be the risk adjusted stock price for a stock given the information from the CAPM model at the end of the period (i.e., either month or year) and P_0 be the price of the stock at the beginning of the period then

$$\tilde{R}_j = \frac{\tilde{P}_e - P_0}{P_0} \tag{4}$$

where R_j is the required rate of return under market equilibrium.

- If the CAPM model holds

$$
\begin{aligned}
E\left[R_j\right] = R_f + \beta\left(E\left[R_m\right] - R_f\right) &= \frac{E\left[\tilde{P}_e\right] - P_0}{P_0} \\
1 + R_f + \beta\left(E\left[R_m\right] - R_f\right) &= \frac{E\left[\tilde{P}_e\right]}{P_0} \\
P_0 &= \frac{E\left[\tilde{P}_0\right]}{1 + R_f + \beta\left(E\left[R_m\right] - R_f\right)}.
\end{aligned} \tag{5}
$$

- This approach is called the **Risk Adjusted Discount Rate** (RADR) approach.

- Implementing this approach in the standard present value analysis

$$PV = \sum_{i=1}^{N} \frac{E\left[CF_{t+i}|I_t\right]}{\left(1 + \left[R_f + \beta_j \left\{ \; R_m - R_f \right\} \; \right]\right)^i} \tag{6}$$

where I_t denotes the information set available at time t.

- A slightly different approach is given by the relationship

$$\frac{1}{\beta_j}\left(R_j - R_f\right) = R_m - R_f \tag{7}$$

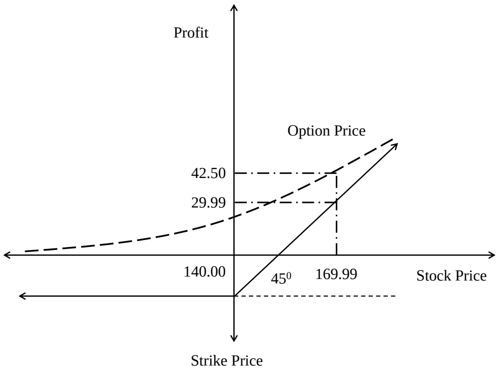
where $1/\beta_j$ measures the certainty equivalent of investment j.

- The modified present value then is written as

$$PV = \sum_{i=1}^{N} \frac{\frac{1}{\beta_j} E\left[CF_{t+i} | I_t\right]}{\left(1 + R_m\right)^i}. \tag{8}$$

Option Pricing Models

- As introduced above, options are derivatives (asset whose value is derived from another asset).



Option Prices for IBM (August 8, 2011)

Strike Price	Call Price	Put Price
120.00	53.80	0.32
140.00	42.50	1.18
150.00	21.80	2.34
160.00	11.40	4.35
170.00	5.45	7.50

- The **strike or exercise price** (X) is the stated price that the investor buys the right to purchase the stock for.
- Market prices for various calls and puts are presented in Table 1.
  - From these prices we see that the call price declines as the strike price increases.
  - On the other side, the put option (the right to sell the stock for the stated strike price) increases as the strike price increases.

- Intuitively, the price of the option is a function of:
  1. the riskiness of the stock (typically estimated as the standard deviation of the daily stock price movement [σ]),
  2. the risk-free interest rate (r_f),
  3. the stock price (S),
  4. the time to expiration (T), and
  5. the strike price X).
- One standard model for valuing the call option is the **Black-Scholes formula**

$$
\begin{gathered}
c = S\ N\left(d_1\right) - X\ e^{-r_f T} N\left(d_2\right) \\
d_1 = \frac{ln\left(\dfrac{S}{X}\right) + r_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\
d_2 = d_1 - \sigma\sqrt{T}
\end{gathered}
\tag{9}
$$

$N\left(z\right)$ is the cumulative standard normal distribution (the probability that a given number is less that z under the standard normal distribution).

- Given the Put-Call parity

$$P = C - S + Xe^{-r_f T} \tag{10}$$

the price of the put option becomes

$$P = Xe^{r_f T} N(-d_2) - SN(-d_1). \tag{11}$$