Lecture XXII: Diversification/Portfolio Analysis

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Expected Value – Variance Analysis

- Most applied work in risk and uncertainty are either applications of **portfolio analysis** or the Capital Asset Pricing Model which is an approach to valuing risk using market prices.
- What are the goals of portfolio analysis – a typical formulation of the portfolio problem is to determine the combination of activities that either maximize expected profit for a given level of risk (typically measured as the variance of income), or minimize risk subject to a required rate of return.

- The typical expected value – variance problem can be written as

$$\begin{aligned} &\min_{z} V\left(\sum_{i=1}^{4} z_i r_i\right) \\ &\text{s.t.} \sum_{i=1}^{4} z_i \bar{r}_i \geq 40,000 \\ &\sum_{i=1}^{4} z_i = 160 \\ &z_i \geq 0 \end{aligned} \tag{1}$$

North Florida Crop Yields and Prices

	Cotton		Peanut		Soybeans		Potato	
Year	Yield (lbs)	Price ($/lb)	Yield (lbs)	Price ($/lb)	Yield (bu)	Price ($/bu)	Yield (cwt)	Price ($/cwt)
2009	723	0.663	3,200	0.207	38	9.50	266	17.7
2008	916	0.504	3,200	0.221	38	8.50	285	16.5
2007	687	0.580	2,700	0.186	24	8.90	287	20.9
2006	789	0.462	2,500	0.173	27	6.25	278	18.8
2005	762	0.480	2,700	0.167	32	5.40	273	14.2
2004	601	0.464	2,800	0.181	34	5.60	308	10.5
2003	610	0.655	3,000	0.185	30	6.90	273	13.8
2002	439	0.440	2,300	0.178	33	5.35	262	14.2
2001	612	0.295	3,050	0.215	29	4.20	310	11.2
2000	480	0.565	2,485	0.300	19	4.45	286	10.5
1999	516	0.425	2,770	0.232	32	4.65	286	11.9

Profit per Acre for North Florida Crops

Year	Cotton	Peanut	Soybean	Potato
2009	299.35	352.40	279.00	2,633.20
2008	281.66	397.20	241.00	2,627.50
2007	218.46	192.20	131.60	3,923.30
2006	184.52	122.50	86.75	3,151.40
2005	185.76	140.90	90.80	1,801.60
2004	98.86	196.80	108.40	1,159.00
2003	219.55	245.00	125.00	1,692.40
2002	13.16	99.40	94.55	1,645.40
2001	0.54	345.75	39.80	1,397.00
2000	91.20	435.50	2.55	928.00
1999	39.30	332.64	66.80	1,328.40
Average	148.40	260.03	115.11	2,026.11
Std. Dev.	105.36	117.68	81	935.03

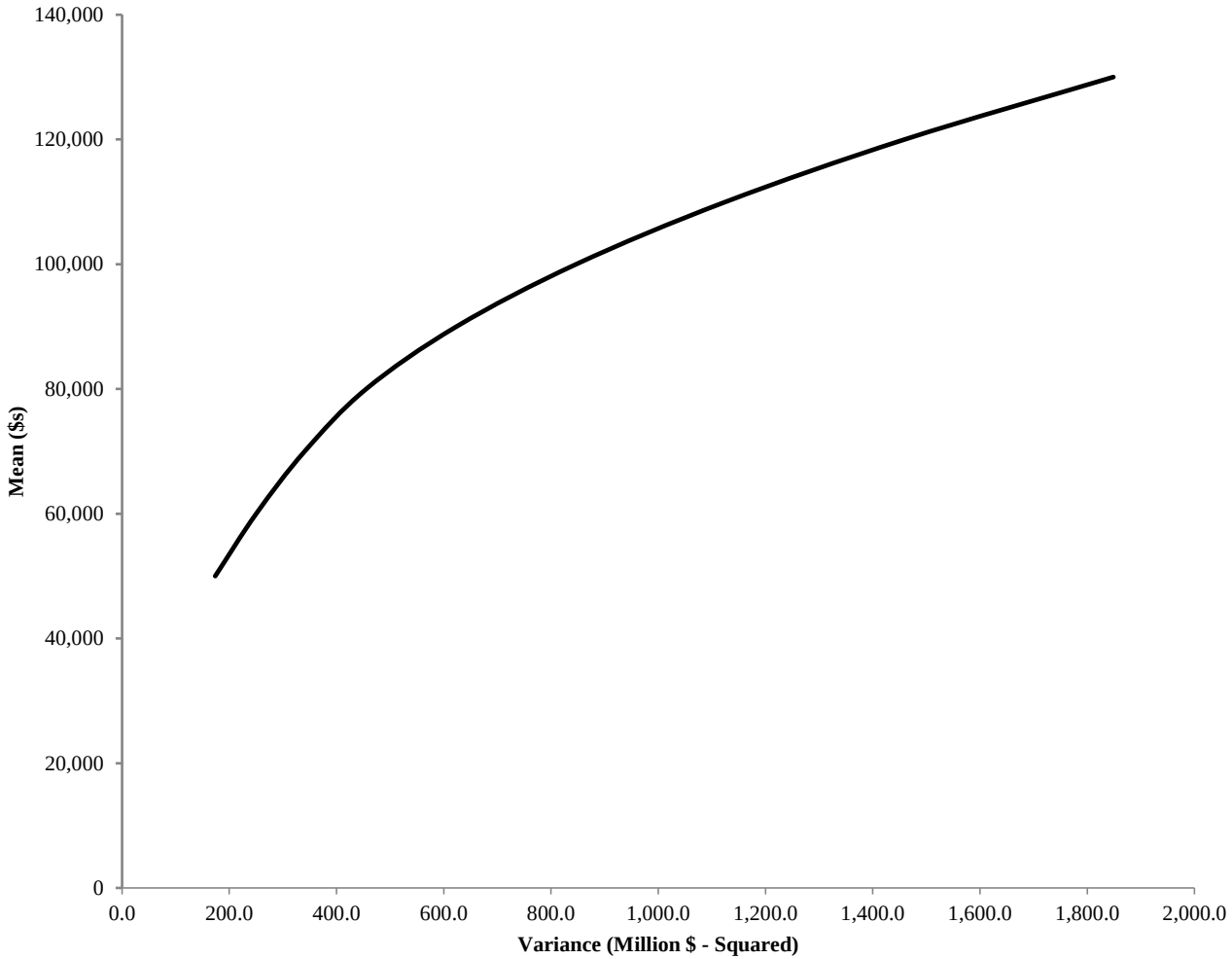
Return Deviations for North Florida Crops

Year	Cotton	Peanuts	Soybeans	Potatoes
2009	150.95	92.37	163.89	607.09
2008	133.26	137.17	125.89	601.39
2007	70.06	-67.83	16.49	1,897.19
2006	36.12	-137.53	-28.36	1,125.29
2005	37.36	-119.13	-24.31	-224.51
2004	-49.54	-63.23	-6.71	-867.11
2003	71.15	-15.03	9.89	-333.71
2002	-135.24	-160.63	-20.56	-380.71
2001	-147.86	85.72	-75.31	-629.11
2000	-57.20	175.47	-112.56	-1,098.11
1999	-109.10	72.61	-48.31	-697.71

Optimal Solutions to the Portfolio Problem

Mean	Variance (mil $\$^2$)	Acres Planted				Acres
		Cotton	Peanut	Soybean	Potato	
50,000	173.8	0.0	97.6	0.0	12.1	109.8
60,000	250.3	0.0	117.1	0.0	14.6	131.7
70,000	340.7	0.0	136.7	0.0	17.0	153.7
80,000	455.4	0.0	138.3	0.0	21.7	160.0
90,000	623.7	0.0	132.6	0.0	27.4	160.0
100,000	847.1	0.0	126.9	0.0	33.1	160.0
110,000	1,125.8	0.0	121.3	0.0	38.7	160.0
120,000	1,459.7	0.0	115.6	0.0	44.4	160.0
130,000	1,848.7	0.0	109.9	0.0	50.1	160.0

Expected Value – Variance Frontier



Using the Mean and Variance Directly

- Remembering our discussion of the loan portfolio from the beginning of this course, consider the variance matrix

$$S = \begin{bmatrix} 11,099.68 & 1,120.60 & 6,739.94 & 63,960.51 \\ 1,120.60 & 13,849.10 & 1,194.86 & -29,440.90 \\ 6,739.94 & 1,194.86 & 6,561.62 & 39,506.75 \\ 63,960.51 & -29,440.90 & 39,506.75 & 874,284.68 \end{bmatrix} \tag{2}$$

- Assume that we want to derive the optimal portfolio of North Florida crops z where z_1 - Cotton, z_2 - Peanuts, z_3 - Soybeans, and $z_4 =$ Potatoes.

- The variance of a particular set of crops can be computed as

$$\begin{gathered}\sigma_p^2(z) = 11,099.68z_1^2 + 2 \times 1,120.60z_1z_2 + \\ 2 \times 6,739.94z_1z_3 + 2 \times 63,960.51z_1z_4 + 13,849.10z_2^2 + \\ 2 \times 1,194.86z_2z_3 - 2 \times 29,440.90 + 6,561.62z_3^2 + \\ 2 \times 39,506.75z_3z_4 + 874,284z_4^2.\end{gathered} \tag{3}$$

- The mean return on the portfolio of crops can be computed as

$$\mu_p(z) = 148.40z_1 + 260.03z_2 + 115.11z_3 + 2,026.11z_4 \tag{4}$$

- The portfolio problem can be written as a form of expected utility formulation

$$\tilde{U} = \mu_p(z) - \frac{\rho}{2}\sigma_p^2(z). \tag{5}$$