Lecture XXI: Expected Utility and Certainty Equivalent

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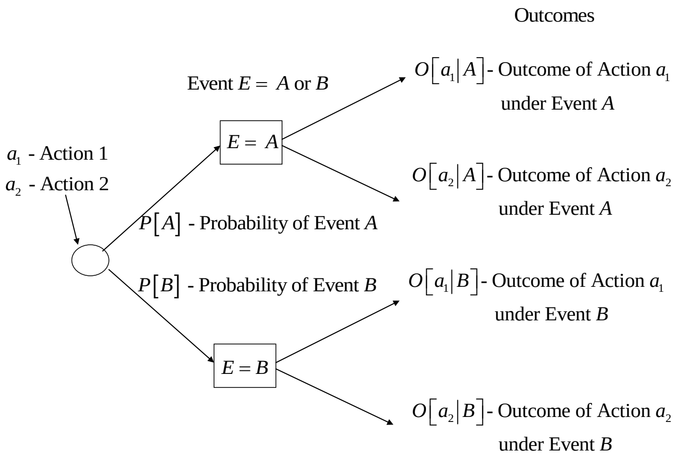
Expected Profit

- Figure 1 develops the decision under risk as the choice between two possible actions (a_1 and a_2).
  - The farmer is choosing between applying 90 pounds of nitrogen per acre or 110 pounds of nitrogen per acre.
  - Regardless of the choice made by the farmer, one of two possible events (E) will occur (either event A or B).
  - The probability that either event will occur is defined by a probability function ($P[A]$ or $P[B]$).
  - The combination of the farmer's actions and possible events yields four possible outcomes such as $O[a_1|A]$ which is the outcome of action a_1 given that event A occurs.
  - Moss 2010 assumes that event A is the case where 30 inches of rain occurs while event B is the case where 35 inches of rain occurs.

Overview of the Risk Problem Moss 2010



Yield and Profit Per Acre

Nitrogen per Acre	Rainfall (inches per season) 30	35
	Corn Yield (Bushels per acre)	
90	41.71	46.46
110	44.30	49.35
	Profit per acre	
90	103.95	116.31
110	109.68	122.80

- How does the farmer decide which alternative to take?
- One alternative is to maximize expected profit.
- Assuming $P[E = A] = 0.60$ and $P[E = B] = 0.40$ the **expected value of profit**(π) for each alternative becomes

$$\begin{array}{l} E[\pi|a_1] = P[E = A]\,116.31 + P[E = B]\,103.95 = 111.37 \\ E[\pi|a_2] = P[E = A]\,122.80 + P[E = B]\,109.68 = 117.55 \end{array} \tag{1}$$

The Expected Utility Hypothesis

- We have appealed to the utility function in our initial development of the capital market.
- The utility function "maps" the consumption of goods into a space of utility or pleasure.
  - In microeconomics, the consumer's problem involves solving for the level of goods that maximize utility subject to a budget constraint.
  - For those of you who have taken a calculus based microeconomic theory course

$$\begin{array}{l} \max_{x_1,x_2} U = x_1^{a_1} x_2^{a_2} \\ \text{s.t. } p_1 x_1 + p_2 x_2 \leq Y \end{array} \quad (2)$$

  - Solving this formulation yields a demand function for x_1 and x_2 which are functions of prices and income

$$\begin{array}{l} x_1^* \left(p_1, p_2, Y\right) \\ x_2^* \left(p_1, p_2, Y\right) \end{array} \quad (3)$$

- In risk analysis, we conceptualize substituting these optimal relationships back into the utility function yielding

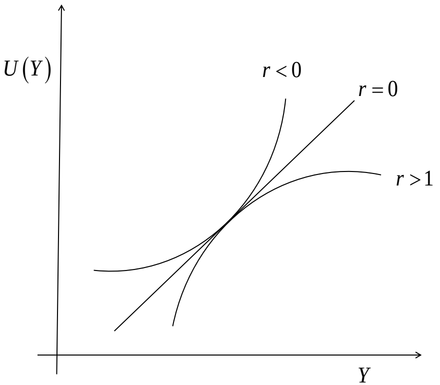
$$V(p_1, p_2, Y) = (x_1^*(p_1, p_2, Y))^{a_1} (x_2^*(p_1, p_2, Y))^{a_2} \tag{4}$$

- Note that utility is now a function of income (and consumption prices). If you give me the level of income, I can tell you what the optimal level of utility is.
- **In this course we do not solve the utility maximization problem**; however, we are interested in the resulting function.
- For example, a frequently used utility function for risk analysis is the power utility function

$$U[Y] = \frac{Y^{1-r}}{1-r} \tag{5}$$

Risk Attitude and Shape of Utility

- This general function gives three types of risk taking behavior: **risk aversion**, **risk taking**, and **risk neutral**



Von Neumann and Morgenstern

- A famous proof in economics is that decision makers take the action that maximizes their expected utility.
- Using the power utility function

$$\begin{aligned} U^* &= P[A]\frac{Y_2^{(1-r)}}{1-r} + P[B]\frac{Y_1^{(1-r)}}{1-r} \\ &= 0.60\frac{113,877^{0.5}}{0.5} + 0.40\frac{98,056^{0.50}}{0.50} = 655.459 \end{aligned} \tag{6}$$

Certainty Equivalent

- The expected utility is also used to define the certainty equivalent – the certain amount that the decision maker would take instead of making the risky gamble.

$$\begin{aligned} U^* &= \frac{Y^{1-r}}{1-r} \\ (1-r)\,U^* &= Y^{1-r} \\ \left[(1-r)\,U^*\right]^{\frac{1}{1-r}} &= Y^* \end{aligned} \quad . \tag{7}$$

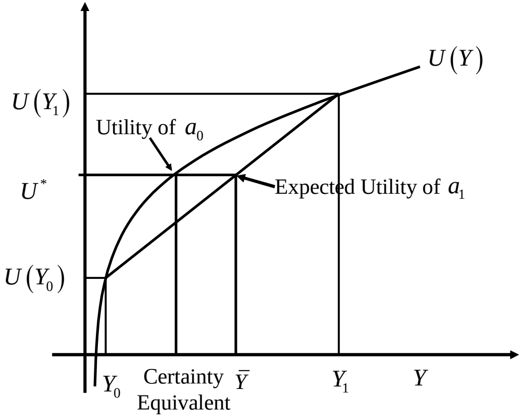
- For the current problem

$$Y^* = [0.50 \times 655.459]^{\frac{1}{0.50}} = 107,407. \tag{8}$$

- The **risk premium**(R_p) is then defined as the difference between the expected value and the certainty equivalent

$$R_p = \bar{Y} - Y^* = 142. \tag{9}$$

Expected Utility Results



Expected Utility, Certainty Equivalent, and Risk Premium

Risk Aversion		90	110
0.40	Expected Utility	1,739.95	1,815.74
	Certainty Equivalent	107,435.06	115,347.55
	Risk Premium	113.42	119.01
0.50	Expected Utility	655.46	679.17
	Certainty Equivalent	107,406.56	115,317.65
	Risk Premium	141.92	148.91
0.75	Expected Utility	72.40	73.70
	Certainty Equivalent	107,335.06	115,242.62
	Risk Premium	213.42	223.94

Expected Utility Under Normality

- Power Utility Function (Log Normal)

$$U^* = \frac{1}{1-r}\exp\left[(1-r)\left(\mu+(1-r)\frac{\sigma^2}{2}\right)\right] \tag{10}$$

- Negative Exponential
  - Utility function

$$U(y) = -\rho\exp(-Y) \tag{11}$$

  - Expected Utility

$$U^* = -\exp\left(-\rho\left(\mu-\frac{\rho}{2}\sigma^2\right)\right). \tag{12}$$