Lecture XX: Other Methods of Investment Analysis

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1. Payback Method
2. Internal Rate of Return
3. Benefit Cost Analysis

Payback Methods

- The **payback method** asks the basic question: How quickly does the alternative pay back its original investment?

	Scenario 1		Scenario 2	
Year	Annual Flow	Accumulated Net Flow	Annual Flow	Accumulated Net Flow
0	-240,854.40	-240,854.40	-188,354.40	-188,354.40
1	69,450.83	-171,403.57	55,172.99	-133,181.41
2	70,145.34	-101,258.23	55,724.72	-77,456.69
3	70,846.79	-30,411.44	56,281.96	-21,174.73
4	71,555.27	41,143.83	56,844.78	35,670.05
5	72,270.81	113,414.64	57,413.23	93,083.28
6	72,993.52	186,408.16	57,987.36	151,070.64
7	73,723.46	260,131.62	58,567.24	209,637.88
8	74,460.69	334,592.31	59,152.91	268,790.79

- The exact payback period for Secnario 1 is computed as

$$PB = 3 + \left(\frac{-30,411.44 - 0}{-30,411.44 - 41.143.83} \right) \times (4 - 3) = 3.425 \quad (1)$$

  of course this degree of precision is somewhat dubious.
- The same estimate for Scenario 2 is 3.373. So both of these investments have nearly identical payback periods.
- Under the payback method, investments with shorter payback periods are preferred to investments with longer payback periods.
- One way to interpret this rule is as extreme risk aversion.

Internal Rate of Return

- The **internal rate of return** is a specialized reformulation of net present value.
- Starting with the simple present value problem

$$NPV = -2,000 + \frac{1,150}{(1.08)} + \frac{1,250}{(1.08)^2} = 126.38. \qquad (2)$$

  - Given what we know about net present, value an increase in the discount rate would result in a lower net present value.
  - Increasing the discount rate to 9 percent would reduce the net present value to \$ 98.30.
  - Intuitively, there exists a discount rate for which the net present value is equal to zero (in this case 13 percent).

Computation of the Internal Rate of Return

- We start by replacing $1/(1+r)$ in Equation 2 with δ and reorder the expression to yield

$$NPV = 1,250\delta^2 + 1,150\delta - 2,000 = 0 \tag{3}$$

so that we can solve for the internal rate of return applying the quadratic formula to Equation 3 yielding

$$\delta = \frac{-1,150 \pm \sqrt{1,150^2 - 4\left(1,250\right)\left(-2,000\right)}}{2\left(1,250\right)} \tag{4}$$
$$= \{\, 0.8859, -1.8060\}$$

The internal rate of return problem in this case (two period) has two solutions.

- To determine the internal rate of return

$$\delta = \frac{1}{(1+r)} \Rightarrow r = \frac{1}{\delta} - 1. \tag{5}$$

  In this case $\delta = 0.8859 \Rightarrow r = 0.1288$ while for $\delta = -1.8060 \Rightarrow -1.5537$.

- One might say: Obviously the first outcome is the investment's internal rate of returninternal rate of return. However, the question remains why?

- Consider a somewhat more complicated cash flow stream – investment I in the preceeding lecture.
- We would determine the internal rate of return by solving

$$NPV = 3,120\delta^4 + 3,210\delta^3 + 3,290\delta^2 + 3,380\delta - 10,500 = 0. \quad (6)$$

This equation has four solutions (by the fundamental theorem of polynomials), but unlike the quadratic its solutions do not have a closed form.

- The numerical results for δ in Equation 6 are

$$\delta = \begin{cases} -1.6056 \\ -0.1693 - 1.5039i \\ -0.1693 + 1.5039i \\ 0.9153 \end{cases} . \quad (7)$$

Clearly, in this example, the feasible delta is 0.9153 implying an internal rate of return of 0.0926.

Internal Rates of Returns for Sample Investments

Investment	IRR	ANPV
I	0.0926	88.09
J	0.0865	60.01
K	0.1015	242.63
K	0.0898	122.51

Benefit Cost Analysis

- Another method for analyzing investment popular in policy and development analysis is the benefit cost approach.
- The **benefit cost** approach essentially divides present value of the revenue stream (or benefits) by the present of required capital outlays (or costs).

$$\mathrm{B/C} = \frac{\displaystyle\sum_{i=0}^{N} \frac{B_i}{(1+r)^i}}{\displaystyle\sum_{i=0}^{N} \frac{C_i}{(1+r)^i}}. \tag{8}$$

- If the benefit to cost ratio is greater than one, the investment is determined to be "profitable" or "beneficial" to the economy.

Present Value Benefit/Cost Analysis

	Investment J		Investment J*		Investment J**	
Year	Cost	Benefit	Cost	Benefit	Cost	Benefit
0	15,000	0	15,000	0	15,000	0
1	0	3,890	0	3,890	0	3,890
2	0	3,610	0	3,610	0	3,610
3	0	3,350	5,000	3,350	250	3,350
4	0	3,100	0	8,500	0	3,233
5	0	2,880	0	2,880	0	2,880
6	0	2,670	0	2,670	0	2,670
PV	15,000	15,277	18,969	19,247	15,198	15,375
B/C	1.018		1.015		1.1012	