Lecture IX: Advance Topics in Present Value Analysis

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Inflation in Net Present Value

- Consistency of definitions is an important dimension of all analysis, but consistency is especially important for net present value analysis because of the possibility of compounding errors.
- We have been treating the net cash flow as a simple aggregate.

$$NPV = -I_0 + \sum_{t=1}^{N} \frac{P_t Y_t - VC_t}{(1+r)^t} \quad (1)$$

  - I_0 is the initial investment,
  - P_t is the price of the output generated by the investment,
  - Y_t is the quantity of output produced by the investment,
  - VC_t is the variable cost needed for production, and
  - r is the discount rate.
- The important consideration here is whether P_t and VC_t are real or nominal.

- Real assumes that the prices series has been adjusted for **inflation** while nominal assumes that no adjustment has been made.
- One approach is estimate P_t and VC_t based on information available at the time the investment is made (i.e., today's price per box of oranges, price per ton of alfalfa, or cost of other inputs used in the production process).
    - Implicitly, using these estimates for future periods assumes that the discount rate is real.
- If we are using a market interest rate (as in the weighted average cost of capital the implicit assumption is that the price series is nominal (i.e., in future or inflated prices).

Fisher's Approximation

- One simplification for the discount rate is Fisher's approximation

$$(1+r) = (1+\tilde{r})(1+i) \approx 1+\tilde{r}+i. \tag{2}$$

  - r is the observed interest rate which includes an adjustment for inflation,
  - $\tilde{r}$ is the real interest rate, and
  - i is the rate of inflation.'

- There are two equivalent ways to adjust for this mismatch.
  - First, we could inflate the net cash flows

$$NPV = -I_0 + \sum_{t=1}^{N} \frac{(P_t Y_t - VC_t)(1+i)^t}{(1+r)^t}. \tag{3}$$

  - Second, we could transform the interest rate to be a **real interest rate**.

$$\begin{gathered}(1+r) = (1+\tilde{r})(1+i) \Rightarrow (1+\tilde{r}) = \frac{(1+r)}{(1+i)} \\ \Rightarrow \tilde{r} = \frac{(1+r)}{(1+i)} - 1\end{gathered} \tag{4}$$

- Consider the numerical example where the weighted average cost of capital is 6.18 % and the inflation rate is 1 %

$$\tilde{r} = \frac{(1+0.0618)}{(1+0.01)} - 1 = 1.0512 - 1 = 0.0512 \quad (5)$$

The adjusted net present valuenet present value formulation then becomes

$$NPV = -I_0 + \sum_{t=1}^{N} \frac{P_t Y_t - VC_t}{\left(\frac{(1+r)}{(1+i)}\right)^t}$$
$$\text{or simply} \quad (6)$$
$$NPV = -I_0 + \sum_{t=1}^{N} \frac{P_t Y_t - VC_t}{(1+\tilde{r})}$$

Adjustments for Taxes

- Taxes considerations can significantly affect investment decisions.
  - At various times the tax code has allowed for "Additional First Year Depreciation" what is typically known as section 179.
  - The "Economic Recovery Tax Act of 1981" accelerated the depreciation schedule for most assets.
  - In agriculture most agricultural land receives a lower tax rate for property taxes – an agricultural exemption typically referred to as the use value.

Salvage Value

- As a beginning point, consider the investment in a machine (say a tractor) that you will use for six years and then sell.
- The sale price of the tractor is its salvage value – say 1,500 for the investment J in the our previous discussion.

$$NPV = -15,000 + \frac{3,890}{(1.08)} + \frac{3.610}{(1.08)^2} + \frac{3,350}{(1.08)^3} + \frac{3,100}{(1.08)^4} + \frac{2,880}{(1.08)^5} + \frac{2,670 + 1,500}{(1.08)^6} = 1,223 \tag{7}$$

- Using a slightly different concept - we can amortize the purchase price minus the present value of the salvage value to determine the annualized net present cost of the equipment purchase

Modified Accelerated Cost Recovery System - Mid Year Convention

Year	3 - Year	5 - Year	7 - Year
1	0.3333	0.2000	0.1429
2	0.4445	0.3200	0.2449
3	0.1481	0.1920	0.1749
4	0.0741	0.1152	0.1249
5		0.1152	0.0893
6		0.0576	0.0892
7			0.0893
8			0.0446

Tax Impact of Cost Recovery Provisions

- The tax code allows the investor to deduct depreciation (or cost recovery of the purchase price of the asset).
- However, we also have to recognize the fact that any income generated by the asset is subject to income tax.
- A fairly complete representation of the investment decision is then

$$NPV = -I_0 + \sum_{t=1}^{N} \frac{\left(P_t Y_t - VC_t\right)\left(1-\tau\right) + M_t \tau}{\left(1 + r\left(1-\tau\right)\right)} \tag{9}$$

  - τ is the firm's marginal tax rate, and
  - M_t is the cost recovery amount (i.e., the value in Table 1).

- An almost complete net present value can then be expressed as

$$NPV = -15,000 + \frac{3,890 \times 0.85 + 15,000 \times 0.15}{(1 + 0.08 \times 0.85)} + \frac{3.610 \times 0.85}{(1 + 0.08 \times 0.85)^2} + \frac{3,350 \times 0.85}{(1 + 0.08 \times 0.85)^3} + \frac{3,100 \times 0.85}{(1 + 0.08 \times 0.85)^4} + \frac{2,880 \times 0.85}{(1 + 0.08 \times 0.85)^5} + \frac{(2,670 + 1,500) \times 0.85}{(1 + 0.08 \times 0.85)^6} = 1,406. \quad (10)$$