Lecture XVIII: Net Present Value and Multiple Projects

Charles B. Moss[1]

[1] Food and Resource Economics Department
University of Florida

January 19, 2018

Net Present Value - The Basic Mechanics

- Moving away from the stylized model of present value, we begin by defining the **net present value** of an investment opportunity as

$$NPV = -I + \sum_{i=1}^{N} \frac{NCF_i}{(1+r)^i} \tag{1}$$

  - NPV is the net present value of the investment opportunity (the wealth created by the investment),
  - I as the cost of the investment item,
  - NCF_i as the net cash flow created by the investment in period i, and
  - r as the weighted average cost of capitalweighted average cost of capital.

- As $I = 0$, $N \rightarrow \infty$ and the cash flows are constant (i.e., $NCF_i = NCF$), the Net Present Value of the investment becomes

$$NPV = \frac{NCF}{r}. \tag{2}$$

- If an investment yields a constant return at the end of each period for N periods, the present value becomes

$$NPV = NCF \left\{ \frac{1 - \left(\frac{1}{1+r} \right)^N}{r} \right\} \tag{3}$$

- Formally, the expression in brackets is referred to as the present value of an ordinary annuity factor of N payments at an interest rate of r $(PVAF_{N,r})$

$$PVAF_{N,r} = \frac{1 - \left(\frac{1}{1+r}\right)^N}{r}. \tag{4}$$

Net Present Value under Varying Discount Rates

Year	Net Cash Flow	Discount Rate				
		0.05	0.06	0.07	0.08	0.09
0	-15,000	-15,000.00	-15,000.00	-15,000.00	-15,000.00	-15,000.00
1	3,250	3,095.24	3,066.04	3,037.38	3,009.26	2,981.65
2	3,250	2,947.85	2,892.49	2,838.68	2,786.35	2,735.46
3	3,250	2,807.47	2,728.76	2,652.97	2,579.95	2,509.60
4	3,250	2,673.78	2,574.30	2,479.41	2,388.85	2,302.38
5	3,250	2,546.46	2,428.59	2,317.21	2,211.90	2,112.28
6	3,250	2,425.20	2,291.12	2,165.61	2,048.05	1,937.87
Total		1,496.00	981.30	491.26	24.36	-420.76

- The computation for the cash flow in Year 1 is

$$\frac{3,250}{(1+0.05)} = 3,095.24 \tag{5}$$

- The computation for the cash flow in Year 2 is

$$\frac{3,250}{(1+0.05)^2} = 2,947.85 \tag{6}$$

- Note that the results indicate that for discount rates less than 8 percent, the net present value is positive.

- We can approximate the discount rate required to make the net present value equal to zero as

$$r^* = \left(\frac{0 - 24.36}{-420.75 - 24.36}\right) \times (0.09 - 0.08) + 0.08 = 0.080547 \tag{7}$$

(or by linearly interpolating between the net present value at 0.09 and 0.08).

- The net present value at 0.080547 is -0.54.
- Here 0.080547 is a good approximation of the internal rate of return which will be more fully developed in a subsequent Lecture.

- Note that the annual cash flows in this example are constant
  - we can rewrite the present value problem as

$$
\begin{aligned}
NPF &= -15,0000 + \sum_{i=1}^{6} \frac{3,250}{(1+r)^i} \\
&= -15,000 + 3,250 \sum_{i=1}^{6} \frac{1}{(1+r)^i} \\
&= -15,000 + 3,250 \times PVAF_{6,r}.
\end{aligned} \tag{8}
$$

- Using the $PVAF_{N,r}$ from Equation 4 we see that for $N = 6$ and $r = 0.06$

$$PVAF_{6,0.06} = \frac{1 - \left(\frac{1}{1+0.06}\right)^{-6}}{0.06} = 4.9173 \tag{9}$$

so the net present value of the investment in Table 1 at a discount rate of 0.06 becomes

$$\begin{aligned} NPV &= -15,000 + 3,250 \times 4.9173 \\ &= -15,000 + 15,981.30 = 981,30 \end{aligned} \tag{10}$$

which conforms to the results given in the table.

Making Comparisons with Net Present Value – Equal Lives

Year	A	NPV_A	B	NPV_B	C	NPV_C
0	-15,000	-15,000.00	-15000	-15,000.00	-15000	-15,000.00
1	3,250	3,037.38	3890	3,635.51	2670	2,495.33
2	3,250	2,838.68	3610	3,153.11	2880	2,515.50
3	3,250	2,652.97	3350	2,734.60	3100	2,530.52
4	3,250	2,479.41	3100	2,364.98	3350	2,555.70
5	3,250	2,317.21	2880	2,053.40	3610	2,573.88
6	3,250	2,165.61	2670	1,779.13	3890	2,592.07
Total		491.26		720.73		263.00

- To motivate the next section a little, consider how we could construct a sequence of returns that (1) are constant and (2) will generate the same Present Value as the annual returns for the decreasing flow investment in Table 2.
- As a starting point

$$\begin{aligned} NPV &= -15,000 + \frac{3,890}{1.07^1} + \frac{3610}{1.07^2} + \frac{3350}{1.07^3} + \frac{3100}{1.07^4} + \frac{2880}{1.07^5} + \frac{2670}{1.07^6} \\ &= -15,000 + K\left(\frac{1}{1.07} + \frac{1}{1.07^2} + \frac{1}{1.07^3} + \frac{1}{1.07^4} + \frac{1}{1.07^5} + \frac{1}{1.07^6}\right) \end{aligned} \quad (11)$$

- The basic question is what is K? We can derive K using the $PVAF_{0.07,6}$ – specifically, let us start with the present value of the decreasing series

$$\frac{3,890}{1.07^1} + \frac{3610}{1.07^2} + \frac{3350}{1.07^3} + \frac{3100}{1.07^4} + \frac{2880}{1.07^5} + \frac{2670}{1.07^6} = 15,720.73 \quad (12)$$

- Therefore

$$K = \frac{15,720.73}{PVAF_{0.07,6}} = \frac{15,720.73}{\frac{1-1.07^{-6}}{0.07}} = \frac{15,720.73}{4.7665} = 3298.14 \tag{13}$$

Year	Decreasing Cash Flow		Constant Cash Flow	
1	3,890	3,635.51	3,298.14	3,082.38
2	3,610	3,153.11	3,298.14	2,880.73
3	3,350	2,734.60	3,298.14	2,692.27
4	3,100	2,364.98	3,298.14	2,516.14
5	2,880	2,053.40	3,298.14	2,351.53
6	2,670	1,779.13	3,298.14	2,197.69
Sum		15,720.73		15,720.73

- Expanding the question a little further – we could ask what annual flow would return the same present value as the original investments?

$$\begin{aligned} NPV &= -15,000 + \frac{3,890}{1.07} + \frac{3610}{1.07^2} + \frac{3350}{1.07^3} + \frac{3100}{1.07^4} + \frac{2880}{1.07^5} + \frac{2670}{1.07^6} \\ &= k\left(\frac{1}{1.07} + \frac{1}{1.07^2} + \frac{1}{1.07^3} + \frac{1}{1.07^4} + \frac{1}{1.07^5} + \frac{1}{1.07^6}\right) \\ &= 720.73 \Rightarrow k = \frac{720.73}{PVAF_{0.07,6}} = \frac{720.73}{4.7665} = 151.21 \end{aligned} \tag{14}$$

	With Initial Investment				Constant W/O Initial Investment	
	Decreasing Cash Flow		Constant Cash Flow			
Year						
0	-15,000.00	-15,000.00	-15,000.00	-15,000.00		
1	3,890.00	3,635.51	3,298.14	3,082.38	151.21	141.32
2	3,610.00	3,153.11	3,298.14	2,880.73	151.21	132.07
3	3,350.00	2,734.60	3,298.14	2,692.27	151.21	123.43
4	3,100.00	2,364.98	3,298.14	2,516.14	151.21	115.36
5	2,880.00	2,053.40	3,298.14	2,351.53	151.21	107.81
6	2,670.00	1,779.13	3,298.14	2,197.69	151.21	100.76
		720.73		720.73		720.73

- So the annualized profit from this investment is $ 151.21.

Multiple Projects

- Much of present value analysis is formulated in a binary decision framework (i.e., to accept the investment or not).
- As a starting point to our discussion, consider the cash inflows for five investment alternatives in Table 5, each of which requires an initial investment of $20,000.
- Table 5 indicates that investments E, F, and G are acceptable at a discount rate of 8 percent.

Cash Flows for Multiple Projects

Year	D	E	F	G	H
1	2,860	4,990	3,860	2,970	2,410
2	2,950	4,430	3,690	3,040	2,580
3	3,040	3,940	3,530	3,110	2,770
4	3,140	3,490	3,380	3,170	2,980
5	3,240	3,100	3,230	3,240	3,200
6	3,340	2,750	3,090	3,320	3,430
7	3,450	2,440	2,950	3,390	3,680
8	3,560	2,170	2,820	3,460	3,950
9	3,670	1,930	2,700	3,540	4,240
NPV at $r = 0.08$	-19.29	1,515.69	765.32	70.67	-425.57

Investment Ranking - Equal Lives

Amount of Capital	Investment	NPV
20,000	E	1,515.69
40,000	E and F	2281.01
60,000	E and F and G	2351.68

- Our ability to rank the investment alternatives in Table 5 using net present value is dependent on the fact that all the investment alternatives have the same **planning horizon**.

Investment Returns for Investments of Unequal Lives

Year	I	J	K	L
0	-10,500	-15,000	-20,000	-20,000
1	3,380	3,890	4,990	3,860
2	3,290	3,610	4,430	3,690
3	3,210	3,350	3,940	3,530
4	3,120	3,100	3,490	3,380
5		2,880	3,100	3,230
6		2,670	2,750	3,090
7			2,440	2,950
8			2,170	2,820
9			1,930	2,700
NPV	291.77	277.41	1,515.69	765.32
$PVAF_{N,0.08}$	3.3121	4.6229	6.2469	6.2469
$ANPV$	88.09	60.01	242.63	122.51

Investment Ranking - Unequal Lives

Amount of Capital	Investment	NPV
10,500	I	88.09
20,000	K	242.63
30,500	I and K	330.72
40,000	K and L	365.14
50,500	I and K and L	453.23
65,500	I and J and K and L	513.24